STUDY OF LINE ASYMMETRIES OF SOLAR OSCILLATIONS ABOVE AND BELOW THE ACOUSTIC CUT-OFF FREQUENCY

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ABSTRACT

A simple model for solar oscillations using a delta function source is developed. From this the power spectrum is computed for different values of the source location and for various values of angular degree l. It is seen that there is marked line asymmetry below the acoustic cut-off frequency ω_{ac}, which corresponds to the asymmetry of bound states in quantum mechanics. The asymmetry reduces above the acoustic cut-off frequency, which corresponds to the asymmetry of scattered states. The asymmetry is found to depend strongly on the source location and on the value of l. The results agree with MDI observations.

1. Introduction

Recent observations of Duvall *et al.* (1993) have indicated that the power spectrum of solar p-modes shows varying amounts of asymmetry. In helioseismology the eigenfrequencies were sometimes determined by assuming that the power spectrum was symmetric and can be fitted by a Lorentzian, which is the case if the solar p-modes were damped harmonic oscillators excited by a stochastic source. This leads to systematic errors in the determination of frequencies. Several authors have studied this problem theoretically and have found that there is inherent asymmetry whenever the waves are excited by a localized source (e.g. Abrams and Kumar, 1996; Gabriel, 1995; Kumar *et al.* 1994; Roxbourgh and Vorontsov, 1995). In this paper, for a spherically symmetric solar model we compute the power spectrum for solar p-modes as a function of l and the source location. This model can be used with the SOHO data Figure 1 to give a worthwhile result for the asymmetry. We hope that studying the line asymmetry will enable us to improve the frequency measurements, find the depth of these localized sources that are responsible for exciting the solar p-modes and also study other aspects of nonadiabaticity which get neglected when asymmetry is ignored.

2. Problem Formulation

In this paper we solve the reduced wave equation (1) in the Cowling approximation (e.g. Gough, 1993) for a delta function source using a solar model. The equation is solved numerically using finite differences subject to the boundary conditions that ψ corresponding to the pressure perturbation is very small far away from the lower turning point and the Sommerfeld radiation condition (4) is applied far away from the upper turning point. This ensures outgoing waves. The resulting system is a complex tridiagonal matrix equation which is solved by a standard routine. This gives the Green's function. Damping is added by making the frequency complex, the imaginary part having the damping coefficient. As a result of this there are two kinds of solutions for which the eigenfrequencies are quantized.(i) bound states for which the frequencies are real and (ii) scattered states for which the frequencies are complex, which is due to the fact that the differential operator becomes non-hermitian.

$$\psi'' + k_r^2\psi = \delta(r - r_s) \tag{1}$$

where $\psi = c_S^2\rho^{\frac{1}{2}}\mathrm{div}\boldsymbol{\xi}$ is related to the pressure perturbation which is a function of one spatial variable r, ρ is the equilibrium density, c_S is the sound speed, r_s is the source location, $\boldsymbol{\xi}$ is the fluid displacement, prime denotes differentiation with respect to that spatial variable, k_r is the vertical component of the local wave number and is given by

$$k_r^2 = \frac{\omega^2}{c_S^2}\left(1 - \frac{\omega_+^2}{\omega^2}\right)\left(1 - \frac{\omega_-^2}{\omega^2}\right) \tag{2}$$

where ω is the wave frequency.

The parameter ω_+ plays the role of the acoustic potential for the acoustic oscillations and ω_- is the potential for gravity modes. Since the frequencies of the p-modes are much higher than ω_-,

$$k_r^2 \approx \frac{\omega^2}{c_S^2}\left(1 - \frac{\omega_+^2}{\omega^2}\right) \tag{3}$$

The acoustic potential has a strong peak just below the photosphere(because of a strong density gradient) and tends to a constant value of 5 mHz in the chromosphere.

The Sommerfeld radiation condition

$$\psi' - ik_{r,0}\psi = 0 \tag{4}$$

is applied far away from the acoustic potential, where $k_{r,0}$ is the value of k_r at the point of application of the radiation condition.

The lower boundary condition $\psi = 0$ is applied far below the lower turning points of the modes.

Equation (1) along with the boundary conditions is solved for $\psi(r)$ from which the pressure perturbation $\delta p(r)$ is computed using equation (5) (e.g. Gough, 1993)

$$\delta p = \left(\frac{gf\rho}{r^3}\right)^{\frac{1}{2}}\psi \tag{5}$$

where g is the acceleration due to gravity and f is given by

$$f = \frac{\omega^2 r}{g} + 2 + \frac{r}{H_g} - \frac{L^2 g}{\omega^2 r} \tag{6}$$

H_g is the scale height of g and $L^2 = l(l+1)$.

ξ_r corresponding to the radial component of the velocity perturbation is computed from the following equation in the Cowling approximation

$$\delta p' + \left(\frac{gL^2}{\omega^2 r^2}\right)\delta p - \left(\frac{gf\rho}{r}\right)\xi_r = 0 \qquad (7)$$

3. Results and Discussions

The eigenfunctions of the bound states oscillate inside the acoustic well and decay outside. The corresponding eigenfrequencies are real and are quantized due to the matching conditions at the boundary of the finite potential. The eigenfunctions of the scattered states oscillate inside the well and are matched to the propogating outgoing damped solutions as a result of the radiation condition. The corresponding eigenfrequencies are complex.

From the Green's function the power spectrum as seen by an observer in the vicinity of the photosphere is computed for the pressure and velocity perturbations. It is seen that the various peaks in the spectra in Figures 2, 3 and 4 exhibit asymmetry. Varying l, the source location r_s and the observing point changes the asymmetry in the power spectrum.

Comparing Figure 1 with Figures 2 and 3 it is seen that the source is located around 50 Km below the photosphere for $l = 200$ as the peaks of Figure 1 and Figure 2 match closely whereas the peaks in Figure 1 and Figure 3 do not match.

From Figure 4 it is seen that for high l the number of bound states decrease and the number of scattered states increase, which is due to the fact that the acoustic potential well becomes shallow thereby allowing only a few bound states. From this it is clear that line asymmetry reduces at high l and may not be as important as it is for low l values.

From the various velocity and pressure power spectra Figures 2, 3 and 4 it is seen that the two asymmetries have the same polarity as opposed to the observations of Duvall *et al.* (1993).

4. Acknowledgements

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Fig. 1: SOI Velocity data for l=200

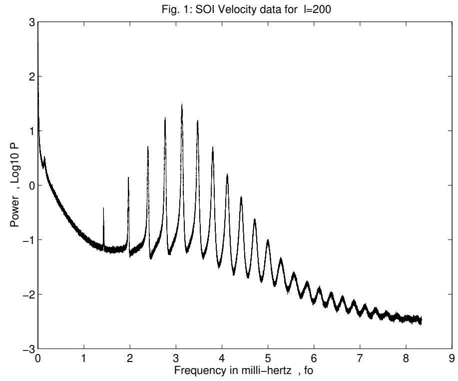

Fig. 2: Power for Source depth = 50 Km and l = 200

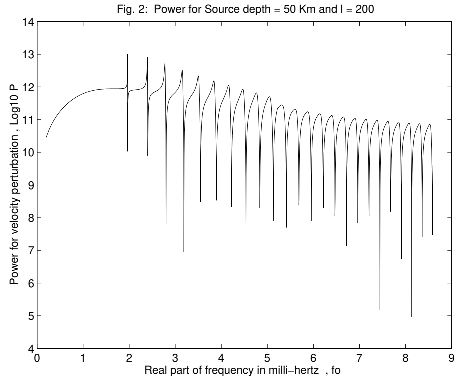

Fig. 3: Power for Source depth = 500 Km and l = 200

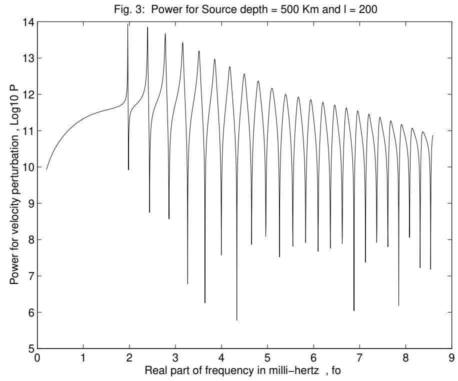

Fig. 4: Power for Source depth = 50 Km and l = 1000