# Determination of the Sun's Seismic Radius from SOHO/MDI

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## ABSTRACT

Dopplergrams from the Michelson Doppler Imager (MDI) instrument on board the SOHO spacecraft have been used to accurately measure frequencies of the Sun's fundamental (f) mode in the medium angular degree range, $l = 88 - 250$. The comparison of these frequencies with the corresponding frequencies of the standard solar models suggests that the apparent photospheric solar radius (695.99 Mm) used to calibrate the models should be reduced by approximately 0.3 Mm. The precise value of the seismologically determined solar radius depends on the description of the subsurface layer of superadiabatic convection. The discrepancy between the 'seismic' and apparent photospheric radii is not explained by the known systematic errors in the helioseismic and photospheric measurements. If confirmed, this discrepancy represents a new interesting challenge to theories of solar convection and solar modeling.

*Subject headings:* Sun: fundamental parameters — Sun: interior — Sun: oscillations — Sun: evolution

## 1. Introduction

We report on the first helioseismic determination of the solar radius using high-precision measurements of oscillation frequencies of the fundamental (f) mode of the Sun, obtained from the MDI experiment on board the SOHO spacecraft (Scherrer et al. 1996). The high quality of the solar Doppler images obtained with the MDI has made it possible to measure reliably the f-mode frequencies in the intermediate angular degree range, $l = 88 - 250$. These observations provide a measure of the intrinsic solar radius for calibrating solar models. Previously, such calibrations were carried out using the apparent photospheric solar radius determined from visual or CCD observations.

Determination of the size of the Sun has been an important astronomical problem for several centuries (e.g. Parkinson et al. 1980). The recent stratospheric and ground-based CCD measurements have provided fairly consistent results for the angular size of the photospheric radius: $959.53 \pm 0.06''$ (Sofia et al. 1994), $959.62 \pm 0.03''$ (Neckel et al. 1995), $959.58 \pm 0.05''$ (Laclare et al. 1996), $959.73 \pm 0.05''$ (Wittmann, 1997). The measurements are corrected for atmospheric shortening, variations of the limb-darkening function and other systematic effects. The error estimates include both random and systematic errors. The measurements of the photospheric radius obtained during the last 25 years from various groups using different instrument types and methods were reviewed by Laclare et al. (1996) who derived the mean value $959.60''$. These results agree well with a standard value of $959.63 \pm 0.10''$, or $695.99 \pm 0.07$ Mm, quoted by Allen (1976) and commonly used for calibrating evolutionary solar models, for computing the Sun's ephemeris in the *Astronomical Almanac* and in other astrophysical applications.

We have compared the observed f-mode frequencies with the corresponding frequencies of several solar models, and found that the model calibration radii have to be changed to match the observed frequencies. The frequencies of the f modes of intermediate angular degree $l$ depend primarily on the force of gravity and on the variation of density in a relatively shallow region below the surface, where these modes propagate. From the asymptotic dispersion relation for the f-mode frequencies, $\omega \simeq \sqrt{gk}$, where $g = GM/R^2$ is the gravity acceleration, $G$ is the gravity constant, $M$ and $R$ are the solar mass and surface radius, $k = L/R$ is the horizontal wave number and $L = [l(l+1)]^{1/2}$, one can easily deduce that $\omega \propto R^{-3/2}$. Using the MDI data we have found that the observed f-mode frequencies are 0.067% higher than those predicted by the standard solar model of Christensen-Dalsgaard et al. (1996). This discrepancy suggests that the solar surface radius used in this model should be accordingly reduced.

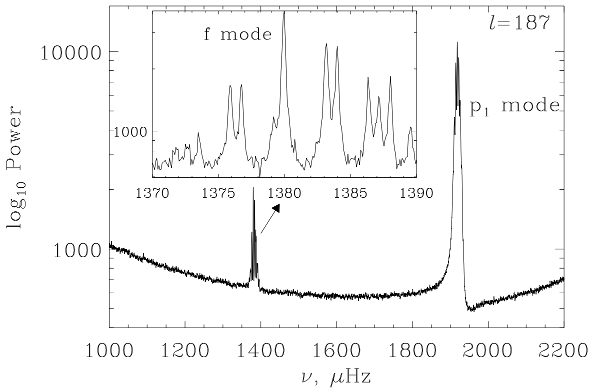


Fig. 1.— Low-frequency part of the $m$-averaged power spectrum of solar oscillations of angular degree $l = 187$ as a function of cyclic frequency $\nu$ ($\omega/2\pi$), obtained from 144 days of MDI Doppler velocity data. The insert shows the f-mode peak for $l = 187$ together with the 'spatial leaks' of modes of adjacent $l$ and $m$.

We study this problem in more detail and argue that the frequencies of the f mode in the observed range of angular degree are determined by the density profile in the region of adiabatic convection beneath the surface. In solar models, this profile is set by the calibration radius which is usually chosen where the local temperature is equal to the Sun's effective temperature, and is assumed to be equal to the photospheric radius, 695.99 Mm. From the analysis of the f-mode frequencies, we suggest the calibration radius of the Christensen-Dalsgaard's standard solar model be reduced by 0.044%, to 695.68 Mm, and show that this 'seismic solar radius' can also improve the agreement between the model sound speed and that inferred by inverting p-mode frequencies.

## 2. Observations

We use a 144-day series of Doppler velocity images taken each minute with the MDI instrument operating in its Medium-$l$ mode (Kosovichev et al. 1997).

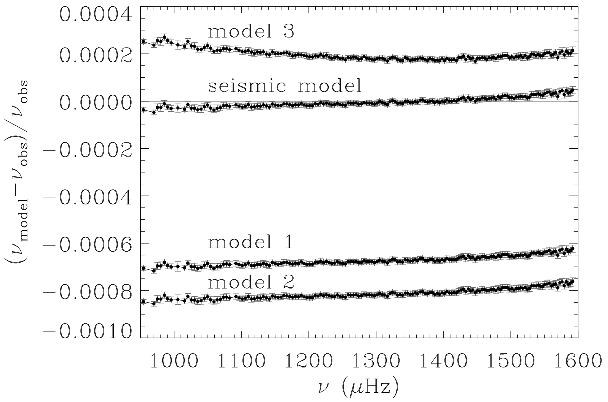


Fig. 2.— Relative differencies between the f-mode frequencies of $l = 88 - 250$ computed for three solar models (Table 1) and the observed frequencies. The 'seismic model' frequencies are obtained by scaling the frequencies of model 1 with factor 1.00067 which corresponds to scaling down the model radius with $(1.00067)^{2/3} \approx 1.00044$. The error bars are $3\sigma$ error estimates of the observed frequencies.

The high stability and low noise of the MDI data allow us to investigate the properties of the fundamental (f) mode, the amplitude of which, in the medium-$l$ range, is substantially lower than the amplitude of acoustic (p) modes. For illustration, in Figure 1, we show a low-frequency part of the MDI power spectrum at $l = 187$. The components of different azimuthal order $m$, split because of rotation and horizontal inhomogeneities, were added together (with an appropriate frequency shift) to provide better signal-to-noise ratio.

The frequencies were determined by simultaneously fitting Lorentzian profiles to the peaks in the amplitude spectra of the f-mode multiplets and approximating the peak frequencies within each $l$-multiplet by the form

$$\nu_{lm} = \nu_l + \sum_{\lambda=1}^{N} a_\lambda^l \mathcal{P}_\lambda^l(m), \qquad (1)$$

where $\nu_l$ is the mean cyclic frequency of the multiplet, $a_\lambda^l$ are the splitting coefficients and $\mathcal{P}_\lambda^l(m)$ are orthogonal polynomials (Schou, 1992).

Parameterization of the frequencies according to Eq. 1 allows us to separate various effects in the mode frequencies. In particular, the odd $a_\lambda^l$ coefficients are used to study the latitudinal variations of the solar rotation (e.g. Kosovichev & Schou, 1997). The even $a_\lambda^l$ coefficients depend on latitudinal inhomogeneities caused by magnetic fields beneath the Sun's surface and by other aspherical perturbations. In this Letter, we focus on the analysis of the mean frequencies of the multiplets, $\nu_l$, which provide information about the radial properties of the solar structure. These frequencies have been measured in the frequency range $950 - 1600$ $\mu$Hz and the corresponding range of $l = 88 - 250$ with precision better than 0.01 $\mu$Hz.

## 3. Determination of the seismic radius

In Figure 2, we compare the observed f-mode frequencies with the theoretical frequencies computed for three different solar models obtained by different authors with different input physics. Some basic characteristics of these models are given in Table 1. For our discussion, it is important to note that models 1 and 2 were calibrated to almost the same radius and based on the standard mixing length theory of convection (MLT), whereas model 3 was calibrated to a smaller radius and based on the Canuto (1990) modification of MLT.

The main component of the relative frequency difference is a constant offset. This shift suggests that the theoretical frequencies are incorrectly scaled. It is known that the oscillation frequencies of stars of the same internal structure but of different mass and radius are scaled with a homology factor $(GM/R^3)^{1/2}$ (e.g. Cox, 1980). Since, for the Sun, $GM$ ($1.32712438 \times 10^{26}$ cm$^3$ s$^{-3}$) is known to high precision from Solar system dynamics (Cohen & Taylor, 1987), the frequency scaling leads to the idea that the value of the solar photospheric radius, $R$, used to calibrate the model is somewhat different from the actual radius.

The last two columns of Table 1 show the average relative frequency shift, $\langle \frac{\nu_{\rm mod} - \nu_{\rm obs}}{\nu_{\rm obs}} \rangle$, and the inferred radius that is required to bring the model frequencies in agreement with the observations, according to the f-mode asymptotic dispersion relation and the homology scaling, $\Delta R/R = -\frac{2}{3}\Delta\nu/\nu$. However, these radius corrections are different even for models 1 and 2 which are calibrated to essentially the same radius.

In order to understand this difference between the radii, we have plotted the density profiles of the solar models in the upper 1.5 Mm of the model (Fig.3). The model photospheric radius is indicated by the stars on the density profiles. Even at the same cal-

Table 1
Solar Models.

| | EOS | Opacity | Diffusion | $R_{\rm mod}$ (Mm) | $\langle \frac{\nu_{\rm mod}-\nu_{\rm obs}}{\nu_{\rm obs}} \rangle \times 10^4$ | $R_{\rm seis}$ (Mm) |
|---|---|---|---|---|---|---|
| model 1[a] | OPAL | OPAL | PM | 695.991 | $-6.74 \pm 0.17$ | $695.678 \pm 0.008$ |
| model 2[b] | SIREFF | OPAL | CGK | 695.975 | $-8.13 \pm 0.18$ | $695.598 \pm 0.008$ |
| model 3[c] | MHD | OPAL92 | no diffusion | 695.630 | $1.95 \pm 0.19$ | $695.720 \pm 0.009$ |

[a]model S of Christensen-Dalsgaard et al. (1996), [b]model 2 of Guzik & Swenson (1997); [c]model 0 of Dziembowski et al. (1994); OPAL refers to the equation of state and opacity tables of Iglesias & Rogers (1996); MHD refers to the equation of state of Mihalas et al. (1990); SIREFF refers to to the equation of state of Swenson et al. (in preparation); MP and CGK refer to the diffusion models of Michaud & Proffitt (1993); and of Cox et al. (1989) respectively; model 3 uses an earlier version of the OPAL92 opacities was used (Iglesias et al. 1992) and the Canuto (1990) modification of the standard MLT formalism for convection; $R_{\rm mod}$ is the photospheric (calibration) radius of the solar models; $R_{\rm seis}$ is the 'seismic' radius obtained from the observed frequencies.

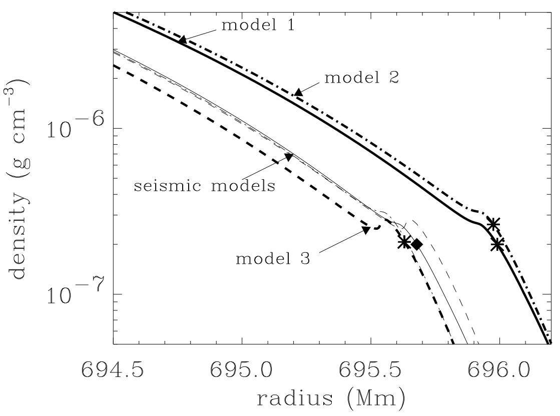


Fig. 3.— Density as a function of the radius near the surface for three solar models listed in Table 1 (thick curves). The stars indicate the values used in the respective models for the photospheric radius, $R_{\rm mod}$, (fifth column of Table 1). The 'seismic models' (thin curves) were obtained by scaling the the corresponding solar models to the seismic radii, $R_{\rm seis}$, indicated in the last column of Table 1. The diamond shows the seismic radius, 695.68 Mm, of model 1.

ibration radius the density profiles are shifted relative to each other. In particular, we notice the radial shift of the adiabatic part of the density profile, which starts about 0.2-0.3 Mm below the photospheric radius. These parts of the curves have almost the same slope because the adiabatic density gradient is essentially independent of the models of convection, low-temperature opacities, equation of state and other complicated physics used to described the upper convective boundary layer and the photosphere. However, the radial position of the adiabatic density profile depends on this physics and, therefore, is different for different models.

It is also important to note that the medium-$l$ f-mode frequencies are predominantly sensitive to the adiabatic part of the convection zone. This is illustrated in Figure 4 which shows the mode energy density as a function of the radius for three modes of $l = 100, 187,$ and 250. The maxima of the energy density are located at approximately 8, 5 and 4 Mm below the photosphere, in the region where convection is believed to be adiabatic.

Therefore, we assume that the seismic radius is determined by the position of the adiabatic density gradient. Then the scaling of the model radii required to match the observed frequencies (the last column of Table 1) brings the adiabatic density profiles together close to the dotted curve in Figure 3, indicated

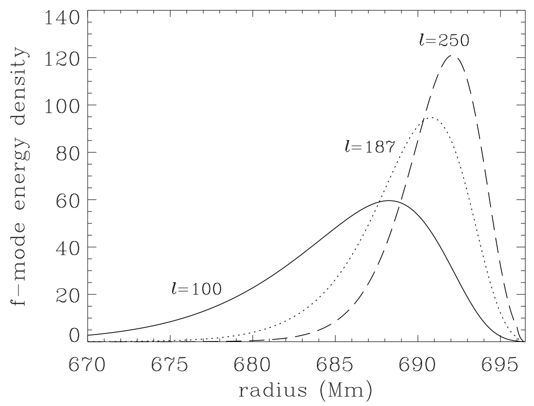


Fig. 4.— The normalized energy density, $\mathcal{E} \equiv \rho[\xi^2 + l(l+1)\eta^2]$, for three f modes of $l = 100, 187$ and 250. Here $\xi$ and $\eta$ are radial and horizontal components of the velocity eigenfunctions, and $\rho$ is density. The mode energy density is normalized so that $\int_0^R \mathcal{E} x^2 dx = 1$, where $x = r/R$.

as 'seismic model'. This dotted curve represents the density of model 1 scaled to the 'seismic radius' of 695.68 Mm, indicated by the diamond.

The numerical results can be also interpreted in terms of the Gough's (1993) asymptotic formula for high-degree f-mode frequencies

$$\omega \simeq [L\frac{GM}{R^3}(1+\epsilon)]^{1/2}, \qquad (2)$$

where $\epsilon = 3(1-\bar{r}/R) - 2L^{-1} + O(L^{-2})$, $\bar{r}$ is the radius where the mode energy density, $\mathcal{E}$, is greatest (see Fig. 4), $L = [l(l+1)]^{1/2}$. The dependence of $\omega$ on the solar structure appears in the term $(1-\bar{r}/R)$ which is $O(L^{-1})$ ($\simeq 10^{-2}$ in our case). This term represents the relative depth of maximum of $\mathcal{E}$, and, therefore, remains unchanged when the solar radius, $R$, is scaled. This conclusion agrees with our numerical results which show variations of $\epsilon$ of the order of $10^{-4}$, or $O(L^{-2})$, among the solar models. Gough also noted that a dependence of f-mode frequencies on the sound speed is only $O(L^{-2})$. This, probably, explains the weak variations with frequency between the model and observed frequencies in Figure 2.

Departures from the asymptotic formula might result also from near surface effects such as magnetic fields or random velocities that have been invoked to explain differences between model and solar frequencies of higher degree f modes (e.g Murawski & Roberts, 1993). These authors have shown that the frequencies could be increased due to the magnetic field and decreased because of the scattering on convective elements. However, using the observed even coefficients, $a^l_\lambda$, of Eq.(1) we have estimated that the mean relative frequency shift caused by the magnetic field does not exceed $2\times10^{-5}$. We have also obtained an upper limit $\sim 10^{-5}$ at $l = 100$ on the frequency shift due to the scattering effect by scaling the shift at $l = 1000$ measured by Bachmann et al. (1995) to $l = 100$ with the inverse mode inertia (cf. Kosovichev, 1995). Generally, contributions of near surface effects to the f-mode frequencies are scaled with the inverse mode inertia which increases by a factor of 20 in our range of $l$. The absence of such a strong variation in Fig.2 indicates that no significant frequency shift in this range of $l$ resulted from the near surface effects.

## 4. Discussion

The analysis of the f-mode frequencies obtained from SOHO/MDI suggests that the value of the solar photospheric radius used to calibrate the standard solar model has to be reduced by approximately 300 km in order to match the model frequencies with the observed frequencies. The f-mode frequencies provide a strict constraint on the density profile $4-10$ Mm beneath the surface, but the precise correction to the calibration radius of a solar model depends on the description of the superadiabatic layer in the model. For the model S of Christensen-Dalsgaard et al. (1996) (model 1 of Table 1), the new calibration radius is approximately 695.68 Mm. The uncertainty due to the statistical errors in the frequency measurements is only 0.008 Mm. However, the systematic error estimated from the deviation of the points of the seismic model in Fig.2 from the zero line can be about 0.03 Mm.

It is intriguing that this correction of the model radius may also explain the sharp decrease near the surface of the relative difference between the squared sound speed, $c^2$, in the Sun, inferred by inversion from the initial GONG and MDI p-mode frequencies, and the standard model (Gough et al. 1996; Kosovichev et al. 1997). In Figure 5, the dashed curve shows the difference between two solar models calibrated to the seismic (695.68 Mm) and the photospheric (695.99 Mm) solar radii. The additional constant sound-speed shift may be related to other surface effects that affect p modes. A similar conclusion follows from the fact that model 3 which has its radius closest to the seismic model is also characterized by much smaller differences in $c^2$ over the whole convective zone than model 1, as it may be seen Fig. 7 of Dziembowski et al. (1994).

Given the high accuracy and consistency of the recent direct measurements of the solar radius and the high precision of the helioseismic results from SOHO/MDI, it seems unlikely that the difference between the photospheric, $695.99 \pm 0.07$ Mm, and the seismic, $695.68 \pm 0.03$ Mm, radii is due to systematic errors. However, we cannot rule out that a modification to the convection theory could extend the adiabatic or superadiabatic density profile of the 'seismic' model up to 695.99 Mm, and, thus, bring the seismic radius into agreement with the photospheric radius. The Canuto (1990) modification of the standard mixing length theory reduces the discrepancy (see Fig.3), but residual is still significant.

If the discrepancy between the seismic and the photospheric solar radii is confirmed it will open interesting perspectives for developing theories of turbulent convection in the Sun and for solar modeling. Measurements of seismic radius changes with the coming increase of the solar activity should help us learn about the influence of magnetic fields on the structure of the Sun's outermost layers.

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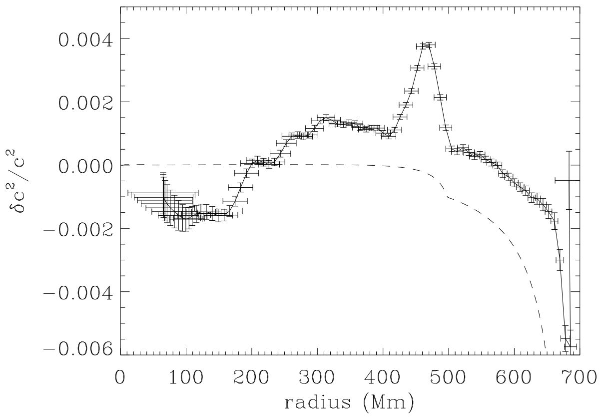


Fig. 5.— The solid curve with the crosses show the relative difference in the squared sound speed between the Sun and model 1 (Kosovichev et al., 1997). The horizontal bars show a characteristic width of the inversion averaging kernels, and the vertical bars are $1\sigma$ error estimates. The dashed curve shows the relative sound-speed difference between a standard solar model calibrated to the seismic radius, 695.68 Mm, and a model calibrated to the photospheric solar radius, 695.99 Mm.

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