Time-Distance Inversion Methods and Results

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Abstract. The current interpretations of the travel-time measurements in quiet and active regions on the Sun are discussed. These interpretations are based on various approximations to the 3-D wave equation such as the Fermat principle for acoustic rays and the Born approximation. The ray approximation and its modifications have provided the first view of the 3-D structures and flows in the solar interior. However, more accurate and computationally efficient approximations describing the relation between the wave travel times and the internal properties are required to study the structures and flows in detail. Inversion of the large three-dimensional datasets is efficiently carried out by regularized iterative methods. Some results of time-distance inversions for emerging active regions, sunspots, meridional flows and supergranulation are presented. An active region which emerged on the solar disk in January 1998, was studied from SOHO/MDI for eight days, both before and after its emergence at the surface. The results show a complicated structure of the emerging region in the interior, and suggest that the emerging flux ropes travel very quickly through the depth range of our observations. The estimated speed of emergence is about 1.3 km/s. Tomographic images of a large sunspot reveal sunspot 'fingers' - long narrow structures at a depth of about 4 Mm, which connect the sunspot with surrounding pores of the same polarity.

Keywords: helioseismology, time-distance tomography, active regions, sunspots, SOHO, MDI

1. INTRODUCTION

Solar oscillations are excited by turbulent convection in the upper convective boundary layer of the Sun. Predominantly excited waves are acoustic (p) and surface gravity waves (f) with oscillation periods of 3-10 min in a wide range of wavenumbers. The combined amplitude of the oscillations is about 200 $\mathrm{m\,s^{-1}}$. The oscillations are usually observed on the solar surface by measuring the Doppler shift of solar absorption lines formed in the lower part of the solar atmosphere.

There are two basic approaches to infer solar properties from the oscillation data. The first approach is to study the resonant properties of the interior by determining the oscillation frequencies of solar normal modes (e.g. Rhodes *et al.* 1997). The second approach is to measure and invert the wave travel times between different points on the surface (Duvall *et al.* 1997). Because of the stochastic nature of solar oscillations substantial spatial and temporal averaging of data is required to measure the frequencies and travel times accurately. These two approaches are complementary: the first is mainly used to infer large-scale properties through the whole Sun, whereas the second has been useful for determining local properties of convective and magnetic structures in the subsurface layers.

In this review, we discuss the basic ideas and procedures for interpretation and inversion of the time-distance data. The first step is in establishing relations between the observed travel-time variations and the internal properties of the Sun (variations of the sound speed, flow velocity and magnetic field). The second step is the inversion of these relations which are typically linear integral equations, to infer the internal properties. The current results of tomographic imaging of convective and meridional flows, an emerging active region, and the structure of a large sunspot are briefly discussed.

2. METHOD OF SOLAR TOMOGRAPHY

In section 2.1, the derivation of the basic tool, the cross correlation function, from a spectrum of mode frequencies is described. In section 2.2, the theoretical connection between the cross correlation function and travel times is derived. In section 2.3, the justification for neglecting damping from the basic theory is presented. In section 2.4, the normal approximation of ray theory is developed. In section 2.5, potential

2.1. Cross-covariance function. Solar seismograms

The basic idea of solar tomography is to measure the acoustic travel time between different points on the solar surface, and then to use these measurements for inferring variations of internal properties such as the sound speed, magnetic field and flow velocities along the wave paths connecting the surface points. This idea is similar to the seismology of the Earth. However, unlike in Earth, the solar waves are generated stochastically by numerous sources in the subsurface layer of turbulent convection. Therefore, the wave travel time is determined from the cross-covariance function, $\Psi(\tau, \Delta)$, of the oscillation signal, $f(t, \boldsymbol{r})$, between different points on the solar surface (Duvall *et al.* 1993):

$$\Psi(\tau,\Delta)=\int_0^T f(t,\boldsymbol{r}_1)f^*(t+\tau,\boldsymbol{r}_2)dt, \tag{1}$$

where Δ is the horizontal distance between the points with coordinates $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$, τ is the delay time, and T is the total time of the observations. In practice, $f(t,\boldsymbol{r})=a(t)d(t,\boldsymbol{r})$, where $d(t,\boldsymbol{r})$ is the observed oscillation signal and $a(t)$ is a tapering function, for which we have adopted a cosine bell that smoothly goes from 1 to 0 in a time $0.05T$. Because of the stochastic nature of excitation of the oscillations, function Ψ must be averaged over some areas on the solar surface to achieve a signal-to-noise ratio sufficient for measuring travel times τ. The oscillation signal, $f(t,\boldsymbol{r})$, is usually the Doppler velocity or intensity variation. A typical cross-covariance function shown in Figure 1 displays several sets of ridges which correspond to the first, second and third and so on bounces of packets of acoustic waves from the surface.

The origin of the multiple bounces is illustrated in Figure 2. Waves originated at point A may reach point B directly (solid curve), or after one bounce at point C (dashed curve), or after two bounces (dotted curve), and so on. Because the sound speed is higher in the deeper layers the direct waves arrive first, followed by the second-bounce and third-bounce waves.

The cross-covariance function represents a solar 'seismogram'. Figure 3 shows the cross-covariance signal as a function of time for a distance of 30 degrees. It consists of three wave packets corresponding to the first, second and third bounces. Ideally, the seismogram should be inverted to infer the structure and flows using a wave theory. However, the application of the wave theory in the three-dimensional case is computationally expensive and has not been developed. Therefore, in practice, as in terrestrial seismology (Aki and Richards, 1980) different approximations are employed, the most simple and powerful of which is the geometrical acoustic (ray) approximation. A first approximation to the wave theory is provided by the Born approximation which is currently under development.

Generally, the observed solar oscillation signal corresponds to velocity or pressure perturbations, and can be represented in terms of normal modes, or standing waves. Therefore, the cross-covariance function can be expressed in terms of normal modes, and then represented as a superposition of traveling wave packets (Kosovichev and Duvall, 1997). This representation is particularly simple for the case of a spherical horizontally uniform stratified layer. In this case the normal oscillation modes are uncoupled and can be represented in terms of individual spherical harmonics, $Y_l^m(\theta,\phi)$. The frequencies of the normal modes,

ω_{nl}, are degenerate with respect to the azimuthal order, m, and depend only on the angular degree, l, and the radial order, n. Representing the observed oscillation signal as a superposition of randomly excited normal modes and applying the spherical harmonic summation theorem the following approximation to the cross-covariance function can be obtained:

$$\Psi(\tau,\Delta) \approx \Re \sum_{nl} a_{nl}\alpha_l P_l(\cos\Delta)\exp\left[-\left(\frac{\omega_{nl}-\omega_0}{\delta\omega}\right)^2 + i\omega_{nl}\tau\right], \quad (2)$$

where a_{nl} is the mode amplitude, $\alpha_l = \sqrt{4\pi/(2l+1)}$, P_l is the Legendre polynomial of degree l, ω_0 is the central frequency of a Gaussian frequency filter applied to the data, and $\delta\omega$ is the characteristic width of this filter. An additional filter is applied to separate the signals from p and f modes. In this paper, we consider only p modes. Time-distance diagnostics based on the f modes is discussed by Duvall and Gizon (2000).

For large l, $P_l(\cos\Delta)$ can be approximated as:

$$P_l(\cos\Delta) \approx \sqrt{\frac{1}{\pi\sin\theta}}\left\{\exp i\left[\left(l+\tfrac{1}{2}\right)\theta - \frac{\pi}{4}\right] + \exp i\left[-\left(l+\tfrac{1}{2}\right)\theta + \frac{\pi}{4}\right]\right\}. \quad (3)$$

The two exponents correspond to waves propagating in the opposite direction between two points. The cross-covariance functions for these waves, $\Psi^+(\tau,\Delta)$ and $\Psi^-(\tau,\Delta)$, determine the reciprocal travel times, τ^+ and τ^-. For a stationary horizontally uniform medium, functions Ψ^+ and Ψ^- are identical. However, in the presence of flow and magnetic fields these functions are different. This may lead to different τ^+ and τ^- times.

An example of the theoretical cross-covariance function for a standard solar model is shown in Figure 4. This model reproduces the observational cross-covariance function very well in the observed range of distances, from 0 to 120 degrees. The theoretical model includes larger distances, up to 180 degrees, which are not accessible currently from observations. We note that a backward propagating ridge originating from the second-bounce ridge at 180 degrees is a geometrical effect due to the choice of the range of the angular distance from 0 to 180 degrees. This is illustrated by the green curve in Figure 2. The waves reaching point F after the reflection at point D propagate more than 180 degrees but are considered as propagating the distance AF which is less than 180 degrees. In the theoretical diagram (Figure 4) one can notice a weak backward ridge between 30 and 70 degrees and at 120 min. This ridge is due to the reflection of acoustic waves from the solar core. It has not been detected in observations.

2.2. Phase and group travel times

By grouping the modes in narrow ranges of the angular phase velocity, $v = \omega_{nl}/L$, where $L = \sqrt{l(l+1)}$, and applying the method of stationary phase, the cross-covariance function can be approximately represented in the form (Kosovichev and Duvall, 1997):

$$\Psi(\tau, \Delta) \propto \sum_{\delta v} a_{\delta v} \cos\left[\omega_0\left(\tau - \frac{\Delta}{v}\right)\right] \exp\left[-\frac{\delta\omega^2}{4}\left(\tau - \frac{\Delta}{u}\right)^2\right], \quad (4)$$

where δv is a narrow interval of the phase speed, $u \equiv (\partial\omega/\partial k_h)$ is the horizontal component of the group velocity, $k_h = L/R$ is the angular component of the wave vector, R is the solar radius. Therefore, the phase and group travel times are measured by fitting individual terms of Eq. (4) to the observed cross-covariance function using a least-squares technique. Details of the analysis of the observational data are given by Duvall (2000).

This technique measures both phase (Δ/v) and group (Δ/u) travel times of the p-mode wave packets. The previous time-distance measurements provided either the group time (Jefferies *et al.* 1994), or the phase times (Duvall *et al.* 1996). It was found that the noise level in the phase-time measurements is substantially lower than in the group-time measurements. Therefore, the phase times are more commonly used. The geometrical acoustic (ray) approximation is often employed to relate the measured phase times to the internal properties of the Sun. More precisely, the variations of the local travel times at different points on the surface relative to the travel times averaged over the observed area are used to infer variations of the internal structure and flow velocities using a perturbation theory.

2.3. Effects of mode damping

In regions of relatively strong magnetic field (sunspot and plages) the acoustic oscillations may experience strong damping which can be due to the interaction with magnetic field, which may lead to transformation of the energy of the acoustic modes into the energy of magnetic modes. Woodard (1997) and Bogdan *et al.* (1998) have suggested that the damping may result in a change in the travel times, which can be different for the waves propagating between the same two points in the opposite directions. The effect of mode damping can be modeled by adding a damping term, $-\gamma_{nl}\tau$, in the square brackets of Eq.(2), where the damping coefficients, γ_{nl}, can be estimated from the mode linewidth in the power spectra. To model strong dissipation which may happen in sunspots we multiplied γ_{nl} obtained for the quiet Sun by 100. Figure 5 shows the cross-covariance functions calculated with this damping and without damping. The mode damping reduces the amplitude of the cross-covariance function but does not affect the phases of this function, and thus does not affect the travel times.

Woodard (1997) presented an example of acoustic rays excited by random point sources on a plane and propagating on the same plane. In this model the cross-covariance function between two sources depends not only on the properties of the medium along the line between these two points but also on the properties outside this line because the outside point sources also contributes to the cross-covariance function. In the vicinity of localized features with strong absorption such as a large sunspot this may affect the measured travel times, and this effect may be different for the reciprocal times, τ^+ and τ^-. However, the estimated effect of ~ 1 s is significantly less than the travel time variations of $\sim 30 - 60$ s which are typically observed in sunspots, and comparable with the noise level. Nevertheless, if this effect takes place then the cross-covariance does not correspond to a model of a simple source-receiver pair assumed in some studies (Jensen *et al.* 1999).

Bogdan *et al.* (1998) attempted to calculate the cross-covariance function for sunspots and other magnetic inhomogeneities by using the mode approach similar to the one used in Sec.2.1. They suggested that the observed difference between τ^+ and τ^- may be explained by a hypothetical phase lag for the incoming waves in the sunspot umbra or by oscillation signals originating from the inner boundary of the sunspot penumbra which could not be distinguished from the umbral oscillations in their analysis of the low-resolution data from the GONG (Braun, 1997). Obviously this spurious phase lag can not appear in high-resolution observations from MDI. Therefore, we conclude that currently there is no evidence that mode damping may significantly affect travel times of acoustic modes, and that the observed variations of the travel times are mostly due to variations in the wave speed and internal flows.

2.4. Ray approximation

In the ray approximation, the travel times are sensitive only to the perturbations along the ray paths given by Hamilton's equations. The variations of the travel time obey Fermat's Principle:

$$\delta\tau = \frac{1}{\omega}\int_\Gamma \delta\boldsymbol{k} d\boldsymbol{r}, \tag{5}$$

where $\delta\boldsymbol{k}$ is the perturbation of the wave vector due to the structural inhomogeneities and flows along the unperturbed ray path, Γ. Using the dispersion relation for acoustic waves in the convection zone the travel-time variations can be expressed in terms of the sound-speed, magnetic field strength and flow velocity (Kosovichev and Duvall, 1997).

The effects of flows and structural perturbations are separated from each other by taking the difference and the mean of the reciprocal travel times:

$$\delta\tau_{\text{diff}} \equiv \tau_+ - \tau_- \approx -2\int_\Gamma \frac{(\boldsymbol{nU})}{c^2} ds; \tag{6}$$

$$\delta\tau_{\text{mean}} \equiv (\tau_+ + \tau_-)/2 \approx -\int_\Gamma \frac{\delta w}{w} S ds, \tag{7}$$

where $\boldsymbol{n}$ is a unit vector tangent to the ray, $\boldsymbol{U}$ is the flow velocity, $S = k/\omega$ is the phase slowness,

$$\frac{\delta w}{w} = \frac{\delta c}{c} + \frac{1}{2}\left[\frac{c_A^2}{c^2} - \frac{(\boldsymbol{kc_A})^2}{k^2c^2}\right] \tag{8}$$

is the relative variation of a wave speed which depends on both the variation of the adiabatic sound speed, $\delta c/c$, and the Alfvén velocity, $\boldsymbol{c_A} = \boldsymbol{B}/\sqrt{4\pi\rho}$, and may be also sensitive to the variations of the acoustic cut-off frequency near the solar surface (Kosovichev and Duvall, 1997).

Magnetic field causes anisotropy of the mean travel times, which, in principle, allows us to separate the magnetic effects from the variations of the adiabatic sound speed (or temperature). So far, only a combined effect of the magnetic fields and temperature variations has been measured reliably.

One-dimensional tests by Kosovichev and Duvall (1997) and two-dimensional numerical simulations by Jensen *et al.* (1999) have shown that equations (6) and (7) provide a reasonable approximation to the travel-time variations of acoustic waves.

2.5. Born approximation. Claerbout's Conjecture.

Generally, a solution to the wave equation

$$\mathcal{L}f + \omega^2 f = F_\omega(\boldsymbol{r}_s) \tag{9}$$

with a point source function $F_\omega(\boldsymbol{r}_s)$ describing solar oscillations with amplitude f can be expressed in terms of the Green's function, $G_\omega(\boldsymbol{r}, \boldsymbol{r}_s)$:

$$f(\boldsymbol{r}, \boldsymbol{r}_s, t) = \int_0^\infty G_\omega(\boldsymbol{r}, \boldsymbol{r}_s) F_\omega(\boldsymbol{r}_s) e^{-i\omega t} d\omega. \tag{10}$$

Using this equation and Eq. (1), the cross-covariance function $\Psi(\Delta, \tau)$ can be calculated for randomly distributed point sources. However, this calculation has not been done except for a spherically symmetrical model (Kosovichev and Duvall, 1997). For this model the cross-covariance function between two points (Eq. 2) is equivalent to a signal from a point source located at one point and received at the other point. The idea that the cross-correlation function would mimic an explosion type signal was also developed in geophysics and lead to 'Claerbout's Conjecture' (see Rickett and Claerbout, 2000):
"By cross-correlating noise traces recorded at two locations on the surface, we can construct the wavefield that would be recorded at one of the locations if there was a source at the other".
This significantly simplifies the interpretation of the time-distance measurements and allows us to take into account wave effects in the theory of travel-time variations. The simplest approach is to assume that the non-spherical perturbations are weak and apply the standard Born approximation. For the spherical case the unperturbed Green's function, G_0 satisfies the following equation:

$$\mathcal{L}_0 G_0 + \omega^2 G_0 = \delta(\boldsymbol{r} - \boldsymbol{r}_s), \tag{11}$$

where $\mathcal{L}_0$ is the unperturbed wave operator. The corresponding equation for the perturbation to the Green's function, G_1, caused by perturbation operator $\mathcal{L}_1$ is:

$$(\mathcal{L}_0 + \mathcal{L}_1)(G_0 + G_1) + \omega^2 (G_0 + G_1) = \delta(\boldsymbol{r} - \boldsymbol{r}_s). \tag{12}$$

Taking the difference between these two equations we obtain:

$$\mathcal{L}_0 G_1 + \omega^2 G_1 = \mathcal{L}_1 (G_0 + G_1). \tag{13}$$

This is known as the Lippman-Swinger equation. The Born approximation is taking by neglecting the G_1 term in the right-hand side of this equation. Then, the solution to the approximate equation can be easily expressed in terms of the unperturbed Green's function and the perturbation described by $\mathcal{L}_1$. The unperturbed Green's function, $\mathcal{G}_0$, can be represented in terms of the normal mode eigenfunctions of a spherically symmetrical solar model. Consequently, the kernels, $K_T(\boldsymbol{r}, \Delta)$, for the travel-time variation:

$$\delta\tau(\Delta) = \int_V K_T(\boldsymbol{r}, \Delta) \frac{\delta w}{w} d\boldsymbol{r} \tag{14}$$

are represented in terms of the mode eigenfunctions similarly to the kernels for normal mode frequency variations (e.g. Kosovichev, 1999). This procedure is described in more detail by Marquering *et al.* (1999), and an example of its application to the Sun is given by Birch and Kosovichev (2000).

One unexpected feature of the travel-time kernels calculated in the Born approximation is that these kernels have zero value along the ray path. This is inconsistent with the idea of Jensen *et al.* (1999) that the wave effects simply make the ray kernels broader. Such kernels are sometimes called 'band-limited ray-path', 'Fresnel-zone' or 'fat-ray' kernels. In fact, the wave kernels show more complicated structure than the ad hoc kernels suggested by Jensen *et al.* One of the important differences between the wave and 'fat'-ray kernels is that the sensitivity along the wave path is not proportional to the inverse local sound speed, but may be significantly different from this, in particular, near the surface (e.g. Stark and Nikolayev, 1993).

3. Inversion methods

Typically, times for acoustic waves to travel between points on the solar surface and surrounding quadrants symmetrical relative to the North, South, East and West directions are measured. In each quadrant, the travel times are averaged over narrow ranges of travel distance Δ. Then, the times for northward-directed waves are subtracted from the times for south-directed waves to yield the time, $\tau_{\rm diff}^{\rm NS}$, which predominantly measures north-south motions. Similarly, the time differences, $\tau_{\rm diff}^{\rm EW}$, between westward- and eastward directed waves yields a measure of east-ward motion. The time, $\tau_{\rm diff}^{\rm oi}$, between outward- and inward-directed waves, averaged over the full annulus, is mainly sensitive to vertical motion and the horizontal divergence. The time, $\tau_{\rm mean}$, which measures sound-speed and magnetic perturbations is also averaged over the full annulus (for more details see Duvall *et al.* 1997, and Kosovichev and Duvall, 1997).

The next step is to determine the variations of the wave speed, w, and flow velocity, $\boldsymbol{U}$, from the observed travel times using equations (6) and (7) or (14). It is assumed that the convective structures and flows do not change during the observations and can be represented by a discrete model. In this model, the 3-D region of wave propagation is divided into rectangular blocks. The perturbations of the wave speed and the three components of the flow velocity are approximated by linear functions of coordinates within each block.

So far, most time-distance inversions are done using the ray approximation. The travel time measured at a point on the solar surface is the result of the cumulative effects of the perturbations in each of the traversed rays of the 3D ray systems. Figure 6 shows, in the ray approximation, the sensitivity to subsurface location for a certain point on the surface. This pattern is then averaged over a certain number of central points (typically, 4-100), and translated for different surface points in the observed area, so that overall the travel times are sensitive to all subsurface points in the depth range 0–20 Mm, in this example.

The sensitivities of the travel times to the parameters of the discrete model are calculated by averaging equations (6) and (7) over the ray systems corresponding to the different radial distance intervals of the data, using approximately the same number of ray paths as in the observational procedure. As a result, two systems of linear equations that relate the data to the sound speed variation and to the flow velocity are obtained, e.g. for the velocity field,

$$\delta\tau_{\mathrm{diff};\lambda\mu\nu} = \sum_{ijk} \boldsymbol{A}^{ijk}_{\lambda\mu\nu} \cdot \boldsymbol{U}_{ijk}, \tag{15}$$

where $\boldsymbol{U}_{ijk}$ are parameters of the discrete model of the 3-D flow field, vector-matrix $\boldsymbol{A}$ maps the structure properties into the observed travel time variations, and indices λ and μ label the location of the central point of a ray system on the surface, and index ν labels the travel distances. These equations are solved by a regularized least-squares technique using the LSQR algorithm (Paige and Saunders, 1982). Jensen *et al.* (1997) suggested to speed up the inversion by doing most of the calculation in the Fourier domain.

The results of test inversions of Kosovichev and Duvall (1997) demonstrate an accurate reconstruction of sound-speed variations and the horizontal components of subsurface flows. However, vertical flows in the deep layers are not resolved because of the predominantly horizontal propagation of the rays in these layers. The vertical velocities are also systematically underestimated by 10-20% in the upper layers. Similarly, the sound-speed variations are underestimated in the bottom layers. These limitations of the solar tomography should be taken into account in interpretation of the inversion results.

The spatial resolution of the time-distance inversions based on the ray kernels and Fresnel-zone kernels was studied in 2D numerical experiments by Jensen *et al.* (1999). It has been found that the resolution kernels for the ray approximation are much sharper than those for the Fresnel-zone approach. However, the shape of the ray-based averaging becomes distorted with depth, which may complicate the interpretation of the inversion results. Also, the structures below the lower turning point of the acoustic ray cannot be resolved with the ray approximation while the Fresnel-zone approach is capable in reproducing some features below the turning point. However, the results become quite distorted. That means that the time-distance data (like other helioseismic data) should not be used for inferring the solar properties below the lower turning points of acoustic waves.

4. Inversion results

Helioseismic tomography has been successfully used to infer local properties of large-scale zonal and meridional flows (Giles *et al.* 1997), convective flows and structures (Duvall *et al.* 1997; Kosovichev and Duvall *et al.* 1997), structure and dynamics of active regions (Kosovichev, 1996), flows around sunspots (Duvall *et al.* 1996). Here we present some results of tomographic inversion for large-scale convective cells ('supergranulation'), meridional flow, sunspot and an emerging active region.

4.1. Quiet-Sun convection and meridional flows

Duvall and Kosovichev (1997) and Duvall *et al.* (1997) used the high-resolution Dopplergrams from the MDI instrument on SOHO to investigate the structure and flows in supergranulation. It was found that, in the upper layers, 2-3 Mm deep, the horizontal flow is organized in supergranular cells, with outflows from the center of the supergranules. The characteristic size of the cells is 20-30 Mm. Comparing with MDI magnetograms, it was found that the cell boundaries coincide with the areas of enhanced magnetic field. These results are consistent with the observations of supergranulation on the solar surface (e.g. Title *et al.* 1989). The supergranulation outflow pattern disappears at the depth of approximately 5 Mm. This suggests that supergranulation is only 5 Mm deep. An alternative interpretation suggesting a depth of 8 Mm was presented by Duvall (1998). The vertical flows determined by the time-distance inversions correlate with the supergranular pattern in the upper layers. Typically, there are upflows in the 'hotter' areas where the sound speed is higher than average, and downflows in the 'colder' areas. The latter are frequently observed in the central part of the outflow regions suggesting that supergranulation is a highly non-stationary phenomenon.

Meridional flows from the equator towards the North and South poles have been observed on the solar surface in direct Doppler-shift measurements (e.g. Duvall, 1979). The MDI observations have provided the first evidence that such flows persist to great depths (Giles *et al.* 1997), and, thus, possibly play an important role in the 11-year solar cycle. The poleward flow can transport the magnetic remnants of sunspots generated at low latitudes to higher latitudes and, therefore, contribute to the cyclic polar field reversal. The meridional flows in the solar interior were detected by the time-distance method. The measured differences between the travel times of acoustic waves propagating poleward and equatorward correspond to the mean meridional flow averaged over the penetration depth of the acoustic waves, which was 4-24 Mm. By using Eq. (6) Giles *et al.* (1997) estimated that the maximum mean speed of the flow is $\simeq 20$ m s^{-1}. They have also found that the flow velocity is almost constant over the observed range of depth.

4.2. Tomography of sunspots and active regions.

In this section we present some previously unpublished results. An important problem in astrophysics is understanding the mechanisms of solar activity. Solar tomography provides a tool for studying the birth and evolution of active regions and complexes of solar activity. In Figure 7, we show the results for the emerging active region observed in January, 1998. This was a high-latitude region of the new solar cycle which began in 1997. Figure 7 shows the distribution of the wave speed variations in a vertical cross-section in the region of the emerging flux and in a horizontal plane at a depth of 18 Mm, for three 8-hour intervals centered at 16:00 UT, January 11, 1998 (a), 00:00 UT, January 12, 1998 (b), and 08:00 UT, January 12, 1998 (c). The perturbations of the magnetosonic speed shown in this figure are associated with the magnetic field and temperature variations in the emerging magnetic ropes. The positive variations are shown in red, and the negative variations are shown in blue.

The top panel (a) shows no significant variations in the region of emergence, which is at the middle of the vertical plane. The MDI magnetogram shown at the top indicates only very weak magnetic field above this region. The middle panel (b) shows a positive perturbation associated with the emerging region. The strongest perturbation in this panel is at the bottom of the observed region. During the next 8 hours (panel c) the perturbation propagated to the top of the box. These results show that the emerging flux propagates very quickly through the upper 18 Mm of the convection zone. We have also analyzed the data for 2-hour intervals and concluded that the emerging flux propagated through the characteristic depth of 10 Mm in approximately 2 hours. This gives an estimate of the speed of emergence ≈ 1.3 km/s. This speed is somewhat higher than the speed predicted by theories of emerging flux. The typical amplitude of the wave-speed variation in the emerging active region is about 0.5 km/s. After the emergence we observed a gradual increase of the perturbation in the subsurface layers, and the formation of sunspots. The observed development of the active region seems to suggest that the sunspots are formed as a result of the concentration of magnetic flux close to the surface.

In Figure 8 we show (in three different projections) an example of the internal structure of a large sunspot observed on June 17, 1998. An image of the spot taken in the continuum is shown at the top. The wave-speed perturbations in the spot are much stronger than in the emerging flux. The typical perturbations range from 0.3 to 1 km/s. At a depth of 4 Mm, a 1 km/s wave-speed perturbation corresponds to a 10% temperature variation (about 2.800 K) or to a 18 kG magnetic field. It is interesting that beneath the spot the perturbation is negative in the subsurface layers and becomes positive in the deeper interior. These data also show 'fingers' - narrow long perturbations which are 4-5 Mm deep and connect to the spot two pores (marked B and C). These pores have the same magnetic polarity as the sunspot. Pore A which has the opposite magnetic polarity is not connected to the spot.

One can suggest that the negative perturbations of the wave speed beneath the spot are, probably, due to the lower temperature. However, the effects of temperature and magnetic field have not been separated in these inversions. The separation these effects is an important problem of solar tomography.

5. Conclusion

Solar tomography, or time-distance helioseismology, provides unique information about three-dimensional structures and flows associated with magnetic field and turbulent convection in the solar interior. This method is at the very beginning of its development. In this paper, we have reviewed some basic interpretations of the time-distance measurements and presented some initial inversion results based on the theoretical ray approximation. Developing wave-form solar tomography is one of the most challenging problems of helioseismology.

Using time-distance tomography, we have been able to measure the structure of supergranulation flows and detect an active region before it appeared on the surface. The inversion results also have shown interesting dynamics of supergranulation, meridional circulation, emerging active regions and the formation of sunspots in the upper convection zone. Further studies of the Sun's interior by time-distance helioseismology will shed light on the mechanisms of solar activity.

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Figure 1. The observational cross-covariance function as a function of distance on the solar surface, Δ, and the delay time, τ. The lowest set of ridges ('first bounce') corresponds to waves propagated to the distance, Δ, without additional reflections from the solar surface. The middle ridge ('second bounce') is produced by the waves arriving to the same distance after one reflection from the surface, and the upper ridge ('third bounce') results from the waves arriving after two bounces from the surface. The backward ridge associated with the second-bounce ridge is due to the choice of the angular distance range from 0 to 180 degrees. The correlation function was derived as the Fourier transform of the power spectrum of solar oscillations in a manner described in Duvall et al. (1993). The power spectrum was calculated from 144 days of MDI data starting in May 1996.

Figure 2. A cross-section diagram through the solar interior illustrating the wave propagation inside the Sun.

Figure 3. The observed cross-covariance signal as a function of time at the distance of 30 degrees.

Figure 4. The theoretical cross-covariance function calculated for a standard solar model.

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Figure 5. The theoretical cross-covariance functions at the distance of 30 degrees calculated without mode damping (solid curve) and with a strong damping (dashed curve)

Figure 6. The regions of ray propagation (shaded areas) as a function of depth, z, and the radial distance, Δ, from a point on the surface. The rays are also averaged over a central area and circular regions on the surface, forming three-dimensional figures of revolution. The dashed lines show the inversion grid.

Figure 7. The sound-speed perturbation in the emerging active region at (a) 16:00 UT, 11-Jan-98, (b) 00:00 UT, 12-Jan-98, and (c) 08:00 UT 12-Jan-98. The horizontal size of the box is approximately 38 degrees (460 Mm), the vertical size is 18 Mm. The panels on the top are MDI magnetogram showing the surface magnetic field of positive (red) and negative (blue) polarities. The perturbations of the sound speed are approximately in the range from -1 to +1 km/s. The positive variations are shown in red, and the negative ones in blue.

Figure 8. The sound-speed perturbation in a sunspot region observed on 20-Jun-98. The horizontal size of the box is 13 degrees (158 Mm), the depth is 24 Mm. The horizontal cut in panels (a) and (b) is located at a depth 21.6 Mm, in panel (c) it is 4.8 Mm deep. The positive variations are shown in red, and the negative ones in blue.