

# Appendix C

## Kernels and Phase speeds

### C.1 Scaling Kernels

Since it is numerically impossible to capture a delta function, we define the anomalies by the following sharply decaying functions,

$$\frac{\delta c^2}{c^2} = 0.1 \left\{ 1 - \frac{1}{1 + \exp[8.2(1 - r)]} \right\}, \quad (\text{C.1})$$

and

$$\frac{\delta a}{a} = - \left\{ 1 - \frac{1}{1 + \exp[8.9(1 - \tilde{r})]} \right\}, \quad (\text{C.2})$$

where  $r = \sqrt{x^2 + y^2 + z^2}$  has units of Mm, and  $(0, 0, 0)$  is the center of the perturbation (it is vertically localized at the level of the photosphere). We use the notation of Gizon & Birch (2002) to describe a source perturbation in equation (C.2), where  $a$  is the strength of the unperturbed source and the deactivated source is located around the point  $(x_0, y_0, z_0)$ , with  $\tilde{r} = \sqrt{(x - x_0)^2 + (y - y_0)^2}$  (units of Mm also) and  $z_0 = -200$  km. Since the  $f$  mode is the diagnostic agent in this case, we assume that the anomaly is essentially 2D in nature. To transform travel-time shifts to kernel magnitudes for the sound-speed perturbation case, consider the function from equation (C.1) applied to equation (3.7). Assuming the kernel varies much slower than

the perturbation, we can rewrite equation (3.7) as:

$$\delta\tau_{\text{mean/diff}}(\mathbf{r}) = K_{\text{mean/diff}}(\mathbf{r}, 0; \Delta) \int \int \int_{\odot} \frac{\delta c^2}{c^2}(\mathbf{r}', z) d\mathbf{r}' dz, \quad (\text{C.3})$$

which when integrated merely becomes the finite volume of the perturbation in equation (C.1). A similar 2D area integration ( $z = -200$  km) is carried out for the source perturbation. Calculating these integrals, the kernel for the sound-speed perturbation (in units of s/Mm<sup>3</sup>) is

$$K_{\text{mean/diff}}(\mathbf{r}, 0; \Delta) = 4.164 \delta\tau_{\text{mean/diff}}(\mathbf{r}), \quad (\text{C.4})$$

and for the  $f$ -mode source kernel (in units of s/Mm<sup>2</sup>),

$$K_{\text{mean/diff}}(\mathbf{r}, 0; \Delta) = -0.3056 \delta\tau_{\text{mean/diff}}(\mathbf{r}). \quad (\text{C.5})$$

## C.2 Phase Speeds

Eleven filters of mean phase-speed  $v$  and width  $\delta v$  are used for different ranges of annulus radii  $\Delta$ , shown in Table C.1. The first column gives the annulus index, the last column gives the center  $t_0$  of the window function used to measure first-bounce travel times (see text).

Table C.1. Annuli and Phase-Speed Filter Parameters

index	mean $\Delta$ (Mm)	$\Delta$ (Mm)	$v$ (km s <sup>-1</sup> )	$\delta v$ (km s <sup>-1</sup> )	$t_0$ (min)
1	6.20	03.7, 04.95, 06.20, 07.45, 08.7	16.40	2.63	19.00
2	8.70	06.2, 07.45, 08.70, 09.95, 11.2	19.28	2.63	19.17
3	11.60	08.7, 10.15, 11.60, 13.05, 14.5	22.26	2.63	20.00
4	16.95	14.5, 15.72, 16.95, 18.17, 19.4	27.24	3.68	25.00
5	24.35	19.4, 21.87, 24.35, 26.82, 29.3	35.73	3.94	27.50
6	30.55	26.0, 28.27, 30.55, 32.82, 35.1	40.06	3.94	29.17
7	36.75	31.8, 34.27, 36.75, 39.22, 41.7	43.25	3.94	30.83
8	42.95	38.4, 40.67, 42.95, 45.22, 47.5	49.20	3.94	33.33
9	49.15	44.2, 46.67, 49.15, 51.62, 54.1	55.80	4.46	35.00
10	55.35	50.8, 53.07, 55.35, 57.62, 59.9	59.25	4.46	36.67
11	61.65	56.6, 59.12, 61.65, 64.18, 66.7	64.37	4.46	38.33