

Appendix D

The scattering coefficients

D.1 Exact solution coefficients

The coefficients A_m and B_m are

$$A_m = \frac{-\left(1 + \frac{\gamma a^2}{2c^2}\right) \frac{k_t}{k} J'_m(k_t R) J_m(kR) + \left(1 - \frac{a^2 k_z^2}{\omega^2}\right) J_m(k_t R) J'_m(kR)}{\left(1 + \frac{\gamma a^2}{2c^2}\right) \frac{k_t}{k} J'_m(k_t R) H_m(kR) - \left(1 - \frac{a^2 k_z^2}{\omega^2}\right) J_m(k_t R) H'_m(kR)} \quad (\text{D.1})$$

and

$$B_m = -\frac{2i}{\pi k R} \left[\frac{k}{k_t} \left(1 + \frac{\gamma a^2}{2c^2}\right) J'_m(k_t R) H_m(kR) - \frac{k^2}{k_t^2} \left(1 - \frac{a^2 k_z^2}{\omega^2}\right) J_m(k_t R) H'_m(kR) \right]^{-1}. \quad (\text{D.2})$$

In equations (D.1) and (D.2), the functions J'_m and H'_m denote the first derivative of J_m and $H_m = H_m^{(1)}$ respectively.

D.2 Useful Integrals

In order to compute scattering amplitudes in the Born approximation, we used (Watson, 1944, chap. 5)

$$\int^x x' J_m^2(kx') dx' = \frac{x^2}{2} [J_m^2(kx) - J_{m-1}(kx) J_{m+1}(kx)] \quad (\text{D.3})$$

and

$$\int^x x' H_m(kx') J_m(kx') dx' = \frac{x^2}{4} [2J_m(kx) H_m(kx) - J_{m-1}(kx) H_{m+1}(kx) - J_{m+1}(kx) H_{m-1}(kx)]. \quad (\text{D.4})$$

D.3 Born approximation coefficients

The coefficients A_m^{Born} and C_m for the Born solution are

$$A_m^{\text{Born}} = -\epsilon \frac{i\pi kR}{4} \left[\left(\gamma + 2 \frac{c^2 k_z^2}{\omega^2} \right) J'_m(kR) J_m(kR) + kR J_m^2(kR) - kR J_{m-1}(kR) J_{m+1}(kR) \right] \quad (\text{D.5})$$

and

$$C_m = -\epsilon \frac{c^2 k^2}{\omega^2} - \epsilon \frac{i\pi kR}{4} \left(\gamma + 2 \frac{c^2 k_z^2}{\omega^2} \right) J'_m(kR) H_m(kR) - \epsilon \frac{i\pi (kR)^2}{8} [2J_m(kR) H_m(kR) - J_{m-1}(kR) H_{m+1}(kR) - J_{m+1}(kR) H_{m-1}(kR)]. \quad (\text{D.6})$$