Appendix D

The scattering coefficients

D.1 Exact solution coefficients

The coefficients A_m and B_m are

$$A_m = \frac{-\left(1+\frac{\gamma}{2}\frac{a^2}{c^2}\right)\frac{k_t}{k}J_m'(k_tR)J_m(kR)+\left(1-\frac{a^2k_z^2}{\omega^2}\right)J_m(k_tR)J_m'(kR)}{\left(1+\frac{\gamma}{2}\frac{a^2}{c^2}\right)\frac{k_t}{k}J_m'(k_tR)H_m(kR)-\left(1-\frac{a^2k_z^2}{\omega^2}\right)J_m(k_tR)H_m'(kR)} \tag{D.1}$$

and

$$B_m = -\frac{2i}{\pi kR}\left[\frac{k}{k_t}\left(1+\frac{\gamma}{2}\frac{a^2}{c^2}\right)J_m'(k_tR)H_m(kR)-\frac{k^2}{k_t^2}\left(1-\frac{a^2k_z^2}{\omega^2}\right)J_m(k_tR)H_m'(kR)\right]^{-1}. \tag{D.2}$$

In equations (D.1) and (D.2), the functions J_m' and H_m' denote the first derivative of J_m and $H_m = H_m^{(1)}$ respectively.

D.2 Useful Integrals

In order to compute scattering amplitudes in the Born approximation, we used (Watson, 1944, chap. 5)

$$\int^x x' J_m^2(kx')\,dx' = \frac{x^2}{2}\left[J_m^2(kx)-J_{m-1}(kx)J_{m+1}(kx)\right] \tag{D.3}$$

and

$$\int^{x} x' H_m(kx') J_m(kx')\, dx' = \frac{x^2}{4}[2J_m(kx)H_m(kx) - J_{m-1}(kx)H_{m+1}(kx) - J_{m+1}(kx)H_{m-1}(kx)]. \quad \text{(D.4)}$$

D.3 Born approximation coefficients

The coefficients A_m^{Born} and C_m for the Born solution are

$$A_m^{\text{Born}} = -\epsilon \frac{i\pi kR}{4}\left[\left(\gamma + 2\frac{c^2 k_z^2}{\omega^2}\right) J_m'(kR)J_m(kR) + kRJ_m^2(kR) - kRJ_{m-1}(kR)J_{m+1}(kR)\right] \quad \text{(D.5)}$$

and

$$\begin{aligned} C_m \;=\; & -\epsilon\frac{c^2k^2}{\omega^2} - \epsilon\frac{i\pi kR}{4}\left(\gamma + 2\frac{c^2k_z^2}{\omega^2}\right) J_m'(kR)H_m(kR) \\ & -\epsilon\frac{i\pi(kR)^2}{8}\left[2J_m(kR)H_m(kR) - J_{m-1}(kR)H_{m+1}(kR) - J_{m+1}(kR)H_{m-1}(kR)\right]. \end{aligned} \quad \text{(D.6)}$$