Chapter 1

Introduction

The Sun, our nearest star, serves as an astrophysical benchmark, contributing to the development of our understanding of stellar evolution, stellar interiors, coronae etc. After the discovery of oscillations on the solar surface almost five decades ago (Leighton, Noyes, & Simon, 1962), there have been significant strides in uncovering various properties of the Sun. Helioseismology (see Christensen-Dalsgaard, 2002; Gizon & Birch, 2005, for reviews), somewhat analogous to geo-seismology, is a collection of methods applied to infer the interior structure and dynamics of the Sun through the study and observation of surface oscillations. The earliest efforts in this field were channeled towards determining the solar stratification and characterizing the nature of the observed modes. With the development of methods such as ring analysis (Hill, 1988), acoustic holography (Lindsey & Braun, 1997), and time-distance helioseismology (Duvall et al., 1993), it became possible in theory to study the fine structure of the Sun. Theory became reality with the advent of observations by the Solar Oscillations Investigations/Michelson Doppler Imager (SOI/MDI) instrument (Scherrer et al., 1995) onboard the Solar and Heliospheric observatory (SOHO) satellite, leading to putative discoveries of the subtle dynamics of local inhomogeneities like sunspots and active regions (e.g. Duvall et al., 1996). This branch of seismology came to be termed as local helioseismology, the study of which shall occupy a significant fraction of this dissertation.

Observations in the Sun begin at the photosphere, where the density is sufficiently low that radiation becomes the predominant energy transport mechanism. This surface is in continual motion due to the interaction, impact and reflection of millions of wave modes. The primary source of wave generation is the intense turbulence present in the convecting uppermost surface layers. In the sun, detected waves that possess diagnostic value are either surface gravity or acoustic modes. While surface gravity modes are constrained to sample only the surface layers, acoustic modes plumb the depths of the solar interior and re-emerge altered by the structure and dynamics of the solar interior. Because the Sun is stratified in a manner that the sound speed monotonically increases from the solar surface to the core, acoustic waves that begin their journey near the photosphere are perpetually refracted away from the center, being redirected towards the surface. This implies that most waves that are directed towards the interior propagate some distance following which they undergo the process of total internal refraction, turning around to return to the surface (e.g. Christensen-Dalsgaard, 2003). The acoustic power spectrum, a measure of the power distribution of the various resonant modes of the Sun, is shown in Figure 1.1.

Because density plummets in the near-surface regions, a large fraction of the acoustic wave spectrum is unable to propagate further, and is reflected, forced to repeat this process of alternate internal refraction and reflection till it is damped out by convective, radiative and other processes. The typical time scale of existence of large wave-length waves is about 5-7 days while small wave-length waves are damped much more quickly and presumably replaced by other waves created by the overturning convective cells (known as granules) at the photosphere. One of the most consequential results to emerge as a side-effect from the theory of solar oscillations is that neutrinos possess three flavours and a non-zero mass. Of less monumental but no doubt interesting and important set of results belong inversions of sunspot dynamics and structure, subsurface solar weather, traveling-wave convection, to name a few.

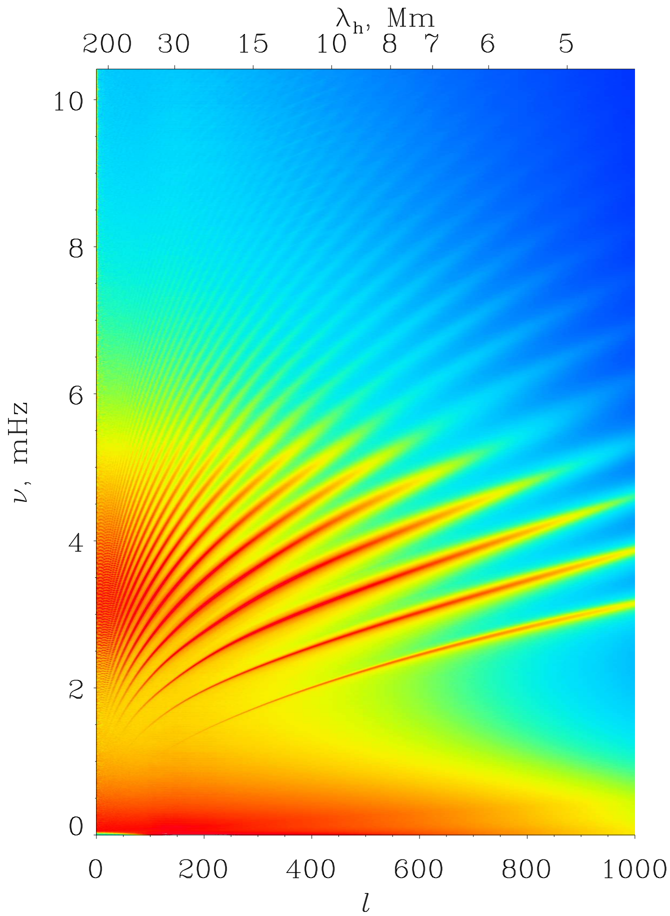

Figure 1.1 The Doppler velocity power spectrum of the Sun as observed by the MDI instrument. The lower horizontal axis is the spherical harmonic degree, l, an alternate measure of the wavelength of the wave, shown on the upper part of the graph as λ_b. The wavelength is expressed in Megameters while l is non-dimensional. The vertical axis is the frequency expressed in milli Hertz. Courtesy: the Solar Oscillations Investigation team.

1.1 Time-distance helioseismology

Time-distance helioseismology in particular has proven to be a very successful and robust method of inferring properties of the solar interior, involving careful manipulations of observations to coax travel times from data. Travel times are the consequential helioseismic metrics in the case of time-distance seismology. Cross correlations of velocity signals (obtained from observations) at pairs of points are computed and averaged in a coherent manner to increase the signal to noise ratio; subsequently these cross correlations are fitted by a standard cross-correlation function or more popularly, a Gabor wavelet (Chou & Duvall, 2000) to obtain travel times (see Figure 1.2).

In a complex and myriad system such as the Sun, the idea of an individual propagating wave makes little sense; a more fitting description may be achieved through the concept of a wave packet. A collection of waves of differing frequencies and wavelengths with one binding condition, that they satisfy a common dispersion relation, is defined as a wave packet. Although many definitions exist for travel times, an approximate (and sometimes misleading; see e.g., Hanasoge, Couvidat, Rajaguru, & Birch, 2007) 'physical' interpretation is the time taken for a wavepacket to travel between two specified points. The travel times of the 'quiet Sun' (where no systematic magnetic activity exists in the region of study) can be computed by tracing rays through a standard model of solar stratification. When travel times deviate in a consistent manner from these standard travel times, there is most likely a perturbation, a local disturbance or inhomogeneity that is the source of these anomalies. Inverse theory then attempts to recover these perturbations from the observed travel-time shifts.

To set up an inverse problem, the connection between perturbations and travel times must be determined; the devices that accomplish this feat are known as kernels. While establishing kernels in the ray approximation (e.g. Giles, 2000) is a relatively easy task, computing them in the limit of finite bandwidth wavepackets (e.g. Birch & Kosovichev, 2000) can be quite difficult. Once the kernel has been determined, the data is averaged in various ways to maximize the signal to noise ratio (SNR); subsequently, the linear inverse problem describing the anomaly of interest can be

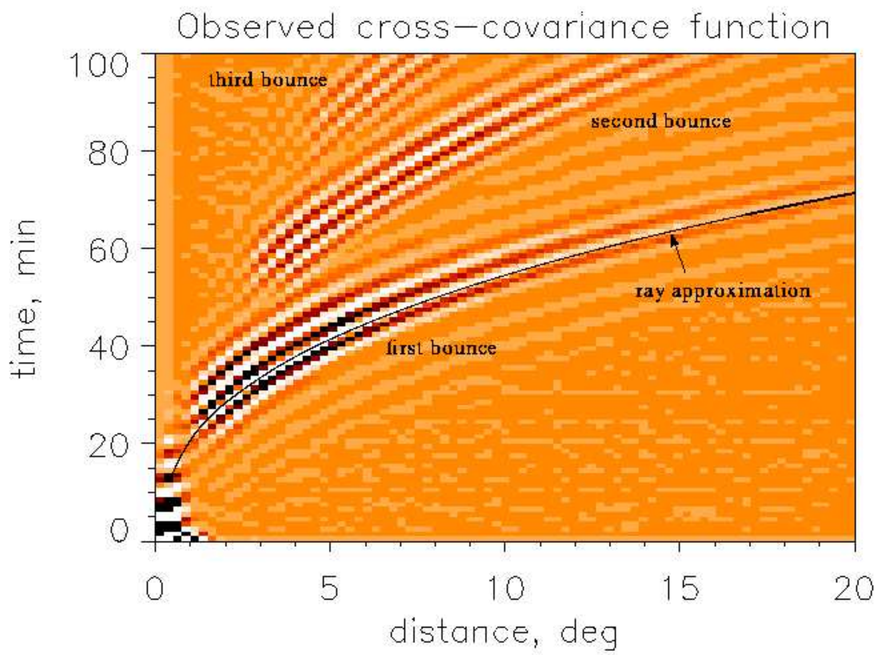

Figure 1.2 A time-distance diagram obtained by analyzing MDI data (Duvall et al., 1997). The ridges seen in the figure correspond to the time (on the vertical axis) taken by wave packets to travel the distance (in degrees) shown on the horizontal axis. The wave packet structure is contained in the ridges.

posed. Inverse problems in helioseismology are invariably ill-conditioned and require considerable regularization to extract solutions. Inverse methods are popular in helioseismology since one can directly seek answers to the question of what lies beneath.

1.2 Magnetic field interactions

Understanding the existence and variability of magnetic activity in the Sun is a problem of wide significance. The ability to predict the solar dynamo, i.e., the regeneration followed by dissipation of magnetic fields in 11 year cycles, is one of the holy grails of solar physics. Another problem of considerable interest and more recently, controversy, relates to the influence of magnetic fields on acoustic waves in the near surface regions of the Sun. In these sub-photospheric magnetic regions, the ratio of magnetic to gas pressure could be very close to unity, leading to the contention that magnetic field effects are systematic and significant. However, due to the difficulties involved in modeling magnetic field effects, most helioseismic analyses tend to ignore these effects, lumping them all into a 'surface term'. Several results that invoke this assumption have come into question and in particular, inversions of the interior structure of sunspots have attracted much debate.

Owing to the intense magnetic field strengths and the inclination of the field in the penumbra (e.g. Schunker et al., 2003), sunspot structure and dynamics inversions (e.g. Duvall et al., 1996; Couvidat, Birch, & Kosovichev, 2006) have been a source of considerable discussion. The very question of how a penumbra is formed around a central umbra in a sunspot is as yet unexplained; current 3D Radiative-Magneto-Hydrodynamic simulations of sunspots are unable to replicate the penumbra. Some of these inversions (Couvidat, Birch, & Kosovichev, 2006) use finite-wavelength descriptions of the acoustic wavefield derived from the approximated constituent equations in the Born limit (Birch & Kosovichev, 2000). Gizon, Hanasoge, & Birch (2006) showed that although the first Born approximation may be valid in regions where the magnetic to gas pressure ratio is much less than unity, its applicability in regions of high magnetic to gas pressure ratios such as sunspots is highly debatable.

Mode conversion (e.g. Barnes & Cally, 2000) is a phenomenon commonly associated with magnetic field induced scattering. From one acoustic mode to another, from acoustic modes to Alfvèn waves and so on, it is estimated that acoustic energy is somewhat redistributed and otherwise lost (in the conversion to Alfvèn waves), contributing perhaps to p-mode absorption observed in sunspots (Braun, 1995). The mode absorption in sunspots detected by Braun (1995) is substantial and until the causal factors are conclusively determined, the relatively simplistic pure acoustic approach in the handling of active regions and sunspots is further threatened.

In terms of taking small steps towards comprehending the influence of magnetic fields on observations, it can be very insightful to estimate the effect of magnetic flux concentrations on waves, the diagnostic agents of the interior. When these interactions are well understood, the next steps will be to use this knowledge to infer interior magnetic structures. Once tomography of the magnetic interior is made possible, views of the interior magnetic field can be used to attempt answers to larger issues such as the existence of the dynamo. However, this is easier said than done because mathematically, it is a difficult proposition to build a clear understanding from the wave equations due to the complex nature of the Lorentz force term and the induction equation. Magnetic forces are tensorial by very nature; consequently parametrizing the effects of competing factors such as the angle of inclination, field strength, size of the flux tube etc. is not an easy task. In fact, even devising a fully magneto-gravito-hydro-static model of a sunspot in the near-surface layers is a highly non-trivial affair.

1.3 The Forward method

One way to address these intricate questions is to numerically compute the interactions of waves with these perturbations. By constructing computational models that mimic the interactions of the solar wave spectrum with various perturbations as closely as possible, it will be possible to lend a clearer interpretation to the observations. This alternative approach to the inverse method (described previously) is known as *forward* modeling. The wavefield is theoretically computed in the presence of various sizes, magnitudes and (perhaps) types of perturbations; shifts in the helioseismic metrics (like travel times) obtained thereof are compared to shifts seen in observational data. These calculations are carried out with the outlook that a close approximation to observations is reason to believe that the structure of the test perturbations are to some extent, representative of reality. However, the uniqueness of the structure is not easy to ensure.

As observations have become increasingly sophisticated, the need for refined forward modeling has become apparent. One reason the forward approach is crucial is that although the resonant mechanical modes of the Sun (the diagnostic agents of helioseismology) have been studied carefully, there are still many curious wave properties neglected in models that may prove significant. For example, finite wavelength effects cast doubt on the validity of the ray approximation in some situations; magnetic fields in the case of sunspots are potentially non-trivial contributors to the wavefield. Because the sound speed in the deep interior is very large, the wavelength, which increases in direct proportion to the sound speed (linear wave propagation), becomes a sizeable fraction of the solar radius. The resolving ability of the wave, commonly estimated to be half to one wavelength for mechanical waves, also decreases, making it impossible to image small-scale dynamics of the deep interior. The fact that waves have very little time (scaling inversely with the sound speed) to sample the structures at these great depths does not aid our cause either. Other wave properties that prove to be a hindrance are geometric spreading and wavefront healing. The former means that the signal is even harder to recover from observations and the latter that the waves are likely to 'forget' that they interacted with a convective cell or some such other structure. In such murky waters (quite literally!), these calculations are very useful because they will likely provide insight into the nature of the signal and the extent of the noise. We can set bounds on detectability and determine the sensitivity of diagnostic agent waves. Although these implications have been known for a while now, a systematic means of investigating such factors have only recently been constructed. Such studies are difficult to conduct by purely analytical means, requiring the introduction of numerical methods to solve the constituent governing equations of wave motion. A significant part of this dissertation attempts to develop and apply techniques specifically to study the solar interior through forward models.

1.4 Results Contained in This Work

The results in this dissertation all fit under the umbrella of wave interactions in the Sun: with flows, thermal asphericities and magnetic fields. Chapter 1 attempts to deliver an overview of helioseismology, the approaches and motivation behind the various problems selected for investigation. In Chapter 2, I describe the numerical methods applied to perform wave calculations in spherical geometry; some validation tests are constructed to ensure the accuracy of the algorithm. Since wave propagation in the Sun is mostly a linear phenomenon and due to the expensive nature of non-linear computations, a linear wave calculation is strongly preferred. However, the near-surface layers of the Sun are so convectively unstable that the stratification in this region must be reconfigured for these simulations to be bounded. A realistic wave excitation mechanism is discussed, the acoustic wave spectrum recovered upon simulations is shown to possess many properties similar to the Sun. The detection of (deep) interior convection has for long, eluded observational efforts. Using 3D non-linear Anelastic Spherical Harmonic (ASH) simulations of interior convection, the question of the detectability of these structures is broached via the ray approximation and wave theory in chapter 2. We determine the signal to noise properties of structures at various depths and estimate the chances of detecting deep convective cells. The focus of this chapter is on the numerical method and basic results with a section on the detectability of flows.

From this point, we make a switch in the geometry, from spherical to Cartesian. In chapter 3, the very useful concept of noise subtraction is introduced, along with a host of other results and validation tests for simulations in Cartesian geometry. Because of the stochastic nature of wave excitation in the Sun, there is a great deal of realization noise in actual data that reduces the signal to noise ratio substantially. In the Cartesian box based computations, we strive to attain similar realization noise properties so as to allow clear interpretations of these simulated results in the context of helioseismology. However, because of finite computational resources, only a limited number of simulations are possible, and this might appear to be a bottleneck. It is shown that with a non-recurrent initial computational cost, the realization noise problem can be considerably mitigated. Applying techniques of time-distance helioseismology, we also show how to recover kernels (used typically to solve inverse problems) from relatively short simulations. Further, a test to verify that the experimental method of extracting kernels from observations is constructed; it shown that indeed, this experimental technique is valid.

Having stated the strong case for pursuing the forward problem, I shall take a tangential step towards theoretical methods of understanding wave interactions with magnetic fields. The validity of the Born approximation, widely used in geophysics and helioseismology, is tested on magnetic fields in chapter 4. Using a simple model of a magnetic flux tube placed in a homogeneous infinite medium, the scattering properties of waves incident on the flux tube are recovered using the Born approximation and independently, an exact solution in the linear limit of small fluctuations. The effect of wavefront healing is also demonstrated.

Continuing with the study of magneto-wave interactions, the scattering matrix and scattering cross-section of a thin flux tube embedded in a realistic polytrope are computed in chapter 5. Scattering measurements in similar situations have been observationally estimated; comparing our theoretical model with actual observations is useful in determining if we possess a sound understanding of this complicated scattering phenomenon. Both numerical and analytical techniques are applied to obtain these scattering properties.