

THEORETICAL STUDIES OF WAVE INTERACTIONS IN THE
SUN

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Abstract

Direct observation of the solar subsurface is impossible due to the high degree of optical scattering by the partially ionized plasma that inhabits the near-surface layers of the Sun. The deepest part of the Sun visible to us, known as the photosphere (also the solar surface), appears as a roiling turbulent radiative magnetized convecting plasma. At first glance, it would seem therefore that subtle questions relating to the subsurface constitution of the Sun seem completely unanswerable and the interior properties unknowable. However, analogous to geoseismology, a great deal can be gleaned about the internal structure and dynamics of the Sun by carefully observing the waves that appear at the solar surface. This has been made possible over the last few decades through the development and application of powerful techniques of helioseismology which combine mathematical rigour and sophisticated guesswork. Analyses of the high quality observations made by the Michelson Doppler Imager (MDI) instrument onboard the Solar and Heliospheric Observatory (SOHO) satellite have led to continuous progress in our ability to infer subtle aspects of the recondite solar interior. This rush of discoveries has brought with it some skepticism and a need to determine whether the diagnostic agents, namely the waves, indeed behave as we expect them to. Moreover, it is instructive to develop an appreciation for the sensitivities of these waves to anomalies at various depths for it tells us what is detectable and how close we are to the detection.

Towards this goal, modeling wave behavior in the Sun using either numerical or analytical techniques is a useful way to proceed. Numerical methods are developed to simulate linear wave propagation in a solar-like stratified medium. Calculations

are performed in spherical and Cartesian geometry, where the former takes into account global, large wavelength, long lived waves and the latter, near-surface short wavelength, short lifetime waves. There are many numerical challenges encountered in these computations: steep density and pressure gradients, convective instabilities, aliasing, boundary conditions etc. that must be dealt with care. Since the problem is computationally intensive, the code is parallelized according to the Message Passing Interface (MPI) standard. Validation procedures to ensure that the reliability of the numerical algorithm are discussed in some detail. Results of analyses of helioseismology on this artificial data are reported and discussed.

The extent of the interaction of magnetic fields with waves is a leading question in helioseismology. Theoretical work that attempts to characterize and quantify these interactions provides a useful complement to the standard numerical approach invoked to address such problems. The Born approximation, a form of perturbation theory, is widely invoked in helioseismic inverse theory. We investigate the Born approximation in the context of magnetic fields and demonstrate its validity and determine the range of applicability. Of more involved theory is the estimation of the wave scattering matrix associated with a thin flux tube embedded in a solar-like medium. Evanescent waves known as jacket modes, which appear due the mathematical incompleteness of the set of resonant mode eigenfunctions are shown to be quite essential in calculating the extent of scatter in the wavefield.

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