Multiscale Analysis of Magnetograms and Dopplerograms

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Abstract

Our scientific objective is to study the ways in which magnetic fields are spread across the surface of the Sun and how they are gathered into local concentrations. We hope to identify various transport mechanisms of fields in the turbulent, convective photospheric plasma as well as the spatial and temporal scales at which these mechanisms operate. This will include exploration of new approaches to the problem of solar turbulent diffusion. We propose to match the turbulent diffusive models to time series of high-resolution Doppler and magnetic MDI images of both solar active regions and quiet Sun areas. Supplementary comparison with contemporaneous correlation tracking and Extreme-Ultraviolet Imaging Telescope data will be important as well.

1 Investigation Plan

The work will focus on two main issues:

Multifractal Analysis and Noise

The objective is to study the structural properties of magnetic and velocity fields at the small scales at which magnetic fields are expected to be concentrated into flux tubes or evolve into more complex forms with a continuous spectrum of scales. We have previously demonstrated that the line-of-sight fields in solar magnetic images possess self-similar scaling (Lawrence et al. 1993, Cadavid et al. 1994, Lawrence et al. 1995). Preliminary studies indicate the same result for Doppler images (Ruzmaikin et al. 1994). Here the situation is complicated by the need to remove the effects of p-mode oscillations; MDI Doppler image time series should be optimal for accomplishing this. From these results we can calculate the filling factor in various regions and can determine the appropriate technique for integration of functions using the multifractal measure. Furthermore, the multifractal spectra of the magnetic and Doppler images are such as to indicate the presence of fields concentrations (singularities). By combining multifractal and two-dimensional wavelet analysis, we can identify locations of the singularities of the multifractal spectrum and can investigate whether they correspond to regions of maximum energy dissipation. The X-ray images from Yohkoh (TRACE) and Extreme-Ultraviolet Imaging Telescope data from SOHO itself taken at the same time will be ideal for this study.

Gaussian noise in digital, photoelectric images has been found to have a characteristic scaling signature. Thus, as a byproduct of the multifractal analysis it is possible to quantify the amount and distribution of such noise in each image.

The Study of Turbulent Diffusion

A second avenue of investigation involves observations of the photospheric velocity field. The goal is to characterize the random components of motions of photospheric magnetic features in quiet Sun and in the enhanced network and plage near active regions. This includes not only estimation of a diffusion coefficient for the motions, but also of possible subdiffusive or superdiffusive properties and effective fractal dimensions of the random walks (Lawrence 1991; Schrijver, et al. 1992; Lawrence & Schrijver 1993; Schrijver 1994; Ruzmaikin et al. 1995).

We will use a self-similar (power-law) form for the velocity field. This approximation is valid in the "intermediate asymptotic" range of scales between granular and supergranular sizes. The idea is to find the two basic parameters of the time-spatial spectrum of this velocity field: the exponent β in the spatial power spectra $E(k) \propto k^{-\beta}$, and the exponent z of the spatial scale dependence of correlation times τ in the non- linear cascade ($\tau \propto k^{-z}$). In the intermediate asymptotic regime these two numbers define the character of turbulent diffusion of passive large-scale scalar quantities, i.e. the exponent in the dependence of the squared distance on time $\langle d^2 \rangle \propto t^h$ (Avellaneda and Majda 1992; Ruzmaikin et al. 1995). For the normal diffusion $h = 1/2$; $h > 1/2$ for an enhanced diffusion; and $h < 1/2$ for a slow diffusion. Although scale-frequency studies in this range have been conducted previously for

Doppler images taken at La Palma, Kitt Peak, Big Bear, and Huairou observatories (Title et al. 1989; Chou et al. 1991) the parameter z, which characterizes the time correlation of every Fourier mode, was not determined in these studies. (In the previous studies the characteristic time was defined to be the correlation lifetime determined by measuring the 1/e width of the temporal autocorrelation function averaged over spatial scales.) In the application to the turbulent diffusion problem, this correlation time carries essential information about the velocity field. The definition of the time parameter and finding it (which is not easy task), together with the characterization of the type of turbulent transport, are the essential differences between these earlier studies and the studies proposed here.

This investigation is rendered especially interesting by the discovery (Avellaneda and Majda 1992) that different regions of the two-dimensional "$\beta - z$ diagram" contain very different kinds of diffusion. For example, when $\beta < 1 - z$, called region I, the long range spatial correlation is weak, and the familiar Fickian diffusion occurs. On the other hand, when $\beta > 3 - 2z$, called region III, long range spatial correlations are very strong. Here the diffusion is independent of the temporal correlation, but is non-stationary and superdiffusive, with an effective diffusion coefficient growing linearly in time. Region II, with $1 - z < \beta < 3 - 2z$, is intermediate and characterized by superdiffusive but stationary transport. The special case of the Kolmogorov turbulence with $\beta = 5/3$ and $z = 2/3$ lies exactly on the boundary between regions II and III.

The type of turbulent diffusion occurring in the solar photosphere has not yet been established. Our goal is to estimate the parameters β and z from a time series of Doppler images and with the help of these parameters determine the type of the solar diffusion (normal, fast, or slow). For the same purpose we will consider high-order correlations in the observed velocity field. If the diffusion is normal the even statistical moments of displacements can be expressed through the second moment. For super or sub diffusion the statistical moments higher than the second are independent quantities, and they have own scaling exponents.

The two parameters β and z enter into many aspects of the transport of quantities advected by the turbulent motions. One such aspect is the fractal dimension of turbulent diffusion fronts, that is, of lines or surfaces advected by the small-scale motions of the fluid, for example the set defined by $B = 0$ or $B = const$. We plan to study two- dimensional cuts across the field distributions, so we are interested in advected line elements. For the Avellaneda and Majda model the fractal dimension of the diffusion front is directly related to the spectral exponents β and z. The fractal dimensions of the level sets can be computed by a two-dimensional box counting or by a correlation procedure with randomized box locations (Lawrence et al., 1993). Another aspect of this calculation is the relative diffusion of pairs of fluid markers. This will yield information analogous to the familiar "Richardson 4/3 law" which holds for hydrodynamic turbulence.

One difficulty with analysis of the Doppler images is that they provide information only about the line-of-sight velocities while it is the transverse velocities that presumably control the motions of fields across the solar surface. To make the connection between these quantities it is necessary to gain information on such structural properties of the motions as aspect ratios of convective cells on different scales. Such information could be accessed by combining analysis of the Doppler images with correlation tracking of the transverse motions.

We hope also to evaluate the "diffusive" contribution to the time evolution of the global field distribution. Differences between quiet Sun and plage behaviors will provide information about the back-reactions of enhanced fields on the plasma motions and may lead to understanding of the stability of plage structures. The only data set usable for this purpose up to now has been that of Schrijver and Martin (1990). Time series of seeing-free MDI magnetograms in which magnetic concentrations can be identified and their motions followed over periods of days (gaps are possible) will be crucial in improving the accuracy of such studies.

2 Data Analysis

To determine the singularity spectrum we will use either a box-counting method (Cadavid, et al. 1994) or a multiplier probability distribution method (Lawrence, et al. 1995) together with a Monte Carlo technique for sampling that improves the counting statistics. The singular points of the multifractal spectrum will be identified on the images by using types wavelet transforms that have been found to be ideally suited for this purpose (Arneodo, Grasseau & Holschneider 1988; Lawrence, Cadavid & Ruzmaikin 1994).

To characterize the turbulent diffusion we plan to use space-time spectra of the background velocity field. To eliminate the p-mode oscillations we have used a subsonic filter, $\omega/k < 7$ km/s. The first step, after the filtering 5 min. oscillations, consists of finding the spectra for two correlators:

$$\langle v(x+s, t+\tau)v(x,t)\rangle, \quad \langle \frac{dv(x+s,t+\tau)}{d\tau} v(x,t)\rangle$$

at $\tau = 0$. The first one is the standard spatial power spectrum. The second spectrum includes the scale distribution of the correlation time. The next step is to find the slopes of the linear parts of the spectra plotted in log-log coordinates. The resulting two numbers define the character of the turbulent diffusion at least on the observed time interval. We then calculate the energy spectrum as a function of ω and $\mathbf{k}$. The energy spectrum of the velocity fields is

$$E(k)d^2k = \int k < dv(k, t+\tau)dv^*(k,t) > dt \tag{1}$$

where $v(k,t)$ is the spatial FFT of the velocity, and the integral is taken over the whole time interval. The exponent β is given by the fit to the straight line $E(k) \propto k^{-\beta}$ in the self-similar part of the spectrum. An estimate for the decorrelation time exponent z is found by use of the relation

$$E(k, \omega = 0) \propto k < v(k,0)v^*(k,0) > \propto k^{-\beta - z}. \tag{2}$$

3 Required Observations

Series of the high-resolution line-of-sight velocity images at intervals of 60 sec or less for up to several hours time. Present ground-based data sets, with attendant seeing distortions, extend over only 4 hours. Quiet and active regions have to be taken separately. In particular, this gives us an opportunity to estimate the back action of the large-scale magnetic field (which is present in active regions) on the diffusion. We are interested in the background continuous spectrum so that it is desirable to get rid of 5 min. oscillations by filtering. The space measurements are needed to avoid the atmospheric distortions intrinsic to the ground based observations and which make the analysis of the image time series difficult. Seeing-free measurements will make feasible the effective filtering of the 5 min. oscillations. Comparison with parallel ground based observations (at San Fernando and La Palma Observatories) of line-of-sight components of the magnetic and velocity fields will give the level and other characteristics of the distortions.

Series of MDI magnetograms at high resolution at intervals of a few hours and covering a given region of the Sun for several days. These should permit the identification of magnetic concentrations from frame to frame and permit tracking of their transverse velocities.

4 Outstanding Problems

Passive scalar diffusion may be a good first approximation to the diffusion of the line-of-sight component of the solar magnetic field. The solar large-scale magnetic field however is not a passive scalar. It is a vector which can affect the turbulent motions. We are in the process of developing theoretical models of magnetic field diffusion which take into account the back action of the fields on the motions. Another extension, similar to a well- known reaction-diffusion approach, is to include the effects of magnetic field reconnections and amplifications. Possible observational implementations will be reported later.

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