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Simulation of Vector-Weighted Binning for the Medium-l Program

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ABSTRACT To optimize the SOI/SOHO Medium-l observing program, we have carried out simulations of a vector-weighted binning scheme using a 20-hour series of full-disk Dopplergrams obtained on 25 January 1996. An optimal set of the Gaussian weights is found, which substantially reduces spatial aliasing in the angular degree range from 0 to 300 while keeping most of the signal of the modes.

1 Introduction

The Medium-l Program of the SOI project on board SOHO is dedicated to continuous monitoring of the internal structure and rotation of the Sun by observing the Doppler velocity signal of solar oscillation modes of low- and intermediate degree l (Scherrer *et al.*, 1996). The Medium-l data are transmitted through the low-rate telemetry (5 kbps) channel that eventually sets the limit to the spatial resolution of the modes. Prior to transmission, the original 1024×1024-pixel CCD measurements are binned on board the spacecraft. The binning can be performed in several ways depending on the scientific objectives of the Program. Kosovichev (1992) has considered spatially non-uniform binning schemes attempting to extend the range of the angular degree of the modes as high as possible keeping sensitivity to all the modes in this range. Alternatively, Gough (1992) suggested focusing on zonal and sectoral modes only, pushing the upper limit to $l = 1500$. These and other possibilities will be explored in the course of the Program. However, the initial goal of the Program is to obtain the most reliable measurements of modes of moderate l, which provide information about the deep interior of the Sun. In some ground-based observations performed with relatively low spatial resolution, the medium-l data are corrupted because of spatial aliasing. Spatial aliasing results in significant problems for measuring parameters of mode lines in the oscillation power spectrum. Milford and Scherrer (1992) have suggested reducing the spatial aliasing by binning the data with Gaussian weights centered on a regular grid (say, of 4×4- or 5×5-pixel spacing) on the CCD with the half-width approximately equal to the space intervals of the grid. Such a 2-D Gaussian filter also reduces solar noise.

2 Gaussian Binning

We have simulated Gaussian-weighted binning using a 20-hour long series of the full disk 1024×1024-pixel Doppler images. The 2D Gaussian filter function for a bin centered at the CCD pixel with coordinates (i_0, j_0) was taken in the form:

$$W(i,j) = 32640 \exp\left(-\frac{(i-i_0)^2}{a^2}\right) \times 32640 \exp\left(-\frac{(j-j_0)^2}{b^2}\right), \tag{1}$$

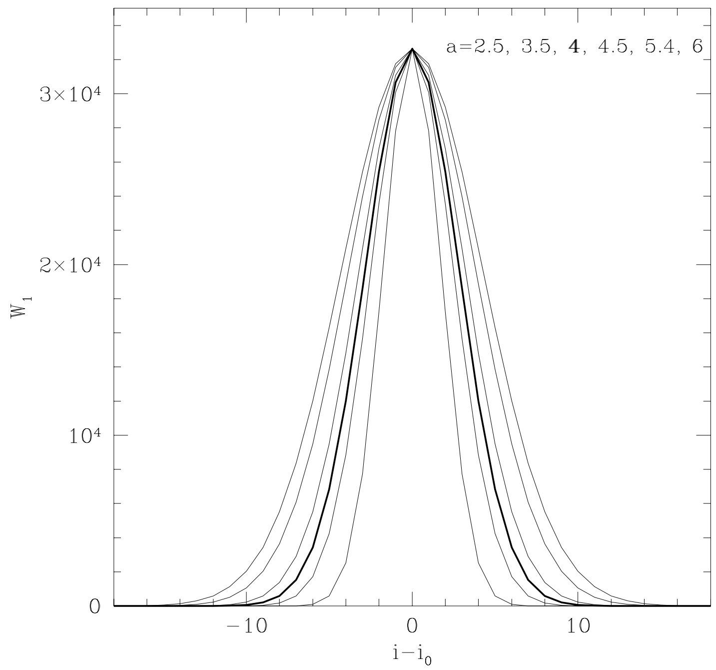

Figure 1: One-dimensional Gaussian weights. The thick curve shows the weights for $a = 4$.

where a and b are parameters of the half-width of the Gaussians, 32640 is the scaling factor. The centers (i_0, j_0) of the bins were uniformly distributed on the 5×5-pixel rectangular grid in the area which covers the full disk up to 0.98 solar radii. We have assumed that $a = b$ in the simulation. The total number of bins was 29804. The simulations were carried out for a=2.5, 3.5, 4, 4.5, 5.4 and 6. For comparison, we have also computed oscillation power spectra for 5×5-pixel box-car averages in the full-disk area, and for 4×4-pixel box-car averages in the central area restricted to $r < 0.8R$ because of the telemetry bandwidth limit. The weighting vectors in each of the two dimensions are listed in the Table 1. They are also shown in Fig. 1.

The data binning reduces the shot noise that must be taken into account in the on-board sum accumulations. We have estimated the coefficient of reduction of the shot noise, C_n, relative to the single pixel value as

$$C_n = \frac{\sqrt{\sum_{i,j} W(i,j)^2}}{\sum_{i,j} W(i,j)}. \tag{2}$$

The coefficient for the chosen values of a together with the noise per bin, computed assuming that the noise accumulated during a 60-sec interval is 12 m/s, is shown in Table 2.

3 Power spectra of the binned data

Power spectra of the binned data were computed using two different techniques: the standard method of remapping the images onto a rectangular grid of heliographic coordinates, and a method of direct projection on eigenmode masks (e.g. Kosovichev, 1992). The results obtained by both techniques are consistent with each other.

Table 1: One-dimensional Gaussian weights

$i - i_0$	a=2.5	a=3.5	**a=4**	a=4.5	a=5.4	a=6
-17					1	10
-16					5	26
-15					14	63
-14				2	39	141
-13				7	99	298
-12			**4**	26	233	597
-11		1	**16**	82	514	1132
-10		9	**63**	233	1057	2029
-9		43	**206**	597	2029	3440
-8	1	175	**597**	1384	3635	5516
-7	12	597	**1526**	2903	6080	8368
-6	102	1727	**3440**	5516	9496	12007
-5	597	4240	**6841**	9496	13848	16298
-4	2523	8841	**12007**	14811	18856	20928
-3	7733	15655	**18597**	20928	23972	25420
-2	17210	23547	**25420**	26789	28456	29207
-1	27813	30081	**30662**	31067	31539	31745
0	32640	32640	**32640**	32640	32640	32640
1	27813	30081	**30662**	31067	31539	31745
2	17210	23547	**25420**	26789	28456	29207
3	7733	15655	**18597**	20928	23972	25420
4	2523	8841	**12007**	14811	18856	20928
5	597	4240	**6841**	9496	13848	16298
6	102	1727	**3440**	5516	9496	12007
7	12	597	**1526**	2903	6080	8368
8	1	175	**597**	1384	3635	5516
9		43	**206**	597	2029	3440
10		9	**63**	233	1057	2029
11		1	**16**	82	514	1132
12			**4**	26	233	597
13				7	99	298
14				2	39	141
15					14	63
16					5	26
17					1	10

Table 2: Shot-noise reduction coefficient, C_n, and the noise per bin, v_n

	a=2.5	a=3.5	**a=4**	a=4.5	a=5.4	a=6
C_n	0.160	0.114	**0.010**	0.0089	0.0074	0.0066
v_n (m/s)	1.92	1.37	**1.20**	1.06	0.89	0.80

The power spectra of zonal modes ($m = 0$) computed for 4×4 and 5×5 box-car averaging are shown in Fig.2. Obviously, the data sampled on the finer spatial grid provide better resolution of p-modes at higher l. Also, the relative amplitude of the aliased modes crossing the original mode ridges is substantially smaller. However, observations with 4×4-pixel averaging can be done only of a smaller portion of the solar disk, thus increasing spatial leakage of adjacent modes, making the mode ridges broader, and thus complicating analysis of individual oscillation modes. With the available data, we cannot estimate the significance of the spatial leakage, a study of which requires much longer time series. However, generally, increased leakage of the adjacent modes is undesirable. Therefore, for the initial Medium-l observations, we suggest a 5×5-pixel grid which covers the full disk.

Figures 3 – 8 show the power spectra of zonal and sectoral modes and of superposition of all the modes (not shifted to $m = 0$) for a=2.5, 3.5, 4, 4.5, 5.4 and 6. As expected increasing a reduces the amplitudes of the aliased ridges, but it also reduces the amplitude of the original modes at higher l. It seems that $a = 4$ represents a reasonable tradeoff between anti-aliasing and high-l cut-off. In this case, we obtain unaliased data up to $l = 300$ (Fig. 5). With this range of p-modes, one should be able to resolve the structure and rotation along the inner 98% of the solar radius, including the HeII ionization zone important for studying solar composition and for testing the equation of state of the solar plasma.

4 Conclusion

For the initial Medium-l Program observations, we suggest the Gaussian vector-weighted data binning on a rectangular grid of 5×5 pixels, in form (1) with $a = 4$. The scheme is expected to provide unaliased spectra of the oscillation modes up to $l = 300$. The shot noise of the binned data is reduced to 0.1 of the single pixel value. If the noise per pixel per 60-sec is 12 m/s then the noise per bin is 1.2 m/s. That means that the on-board sum accumulations should be done in 0.3 m/s precision.

REFERENCES

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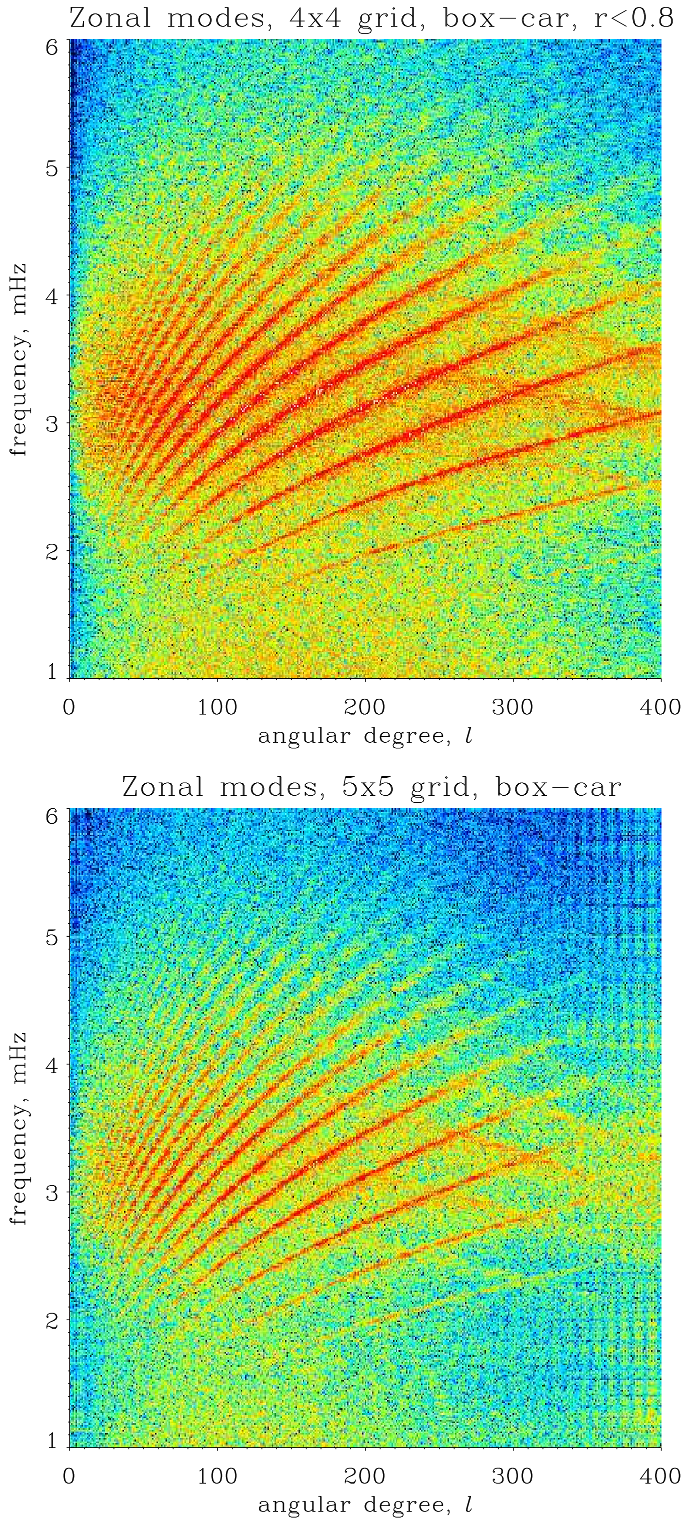

Figure 2: Power spectra of the zonal modes for the 4×4 spacing grid covering the central part of the solar disk ($r < 0.8R$) (top) and for the 5×5 spacing covering the full disk

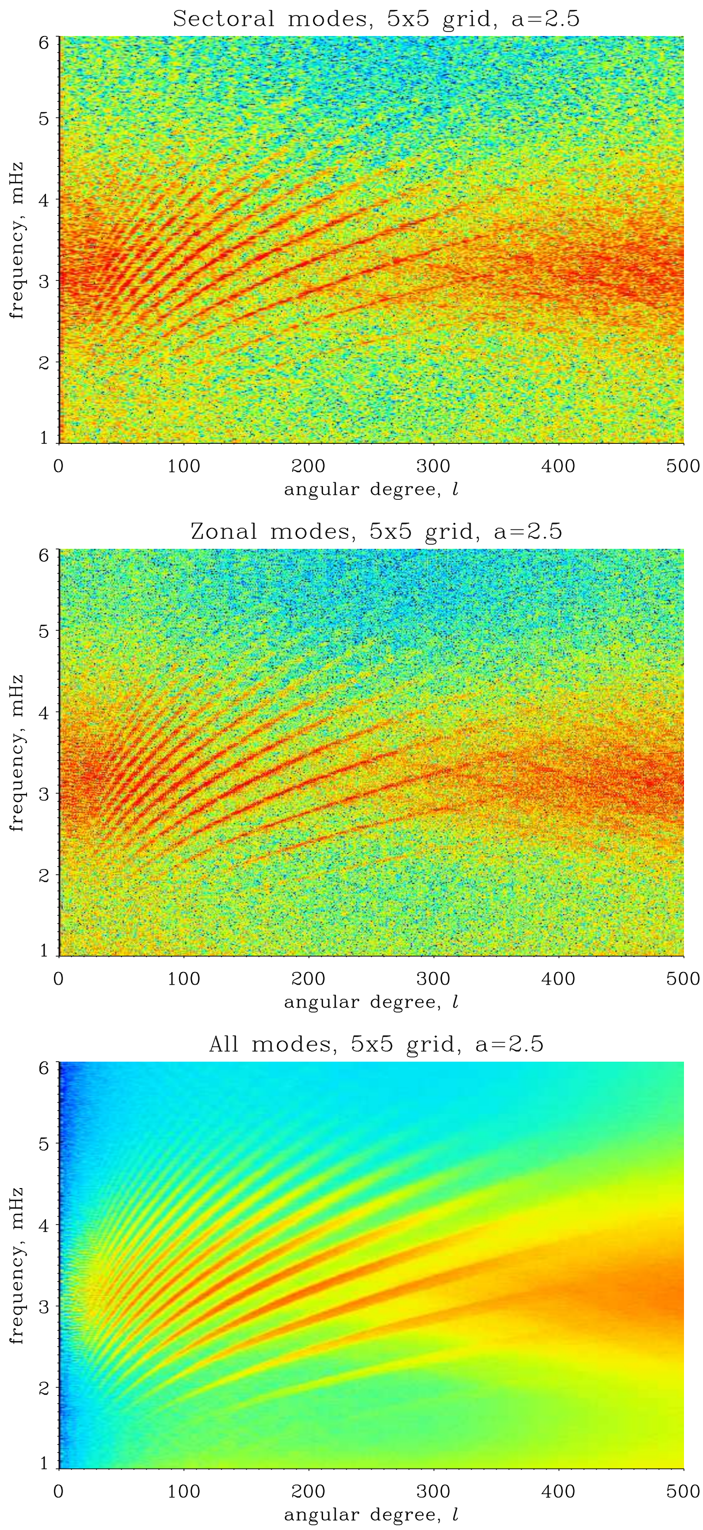

Figure 3: Power spectra of the sectoral and zonal modes and of superposition of all the modes (not reduced to $m = 0$) for Gaussian filter (1) with $a = b = 2.5$.

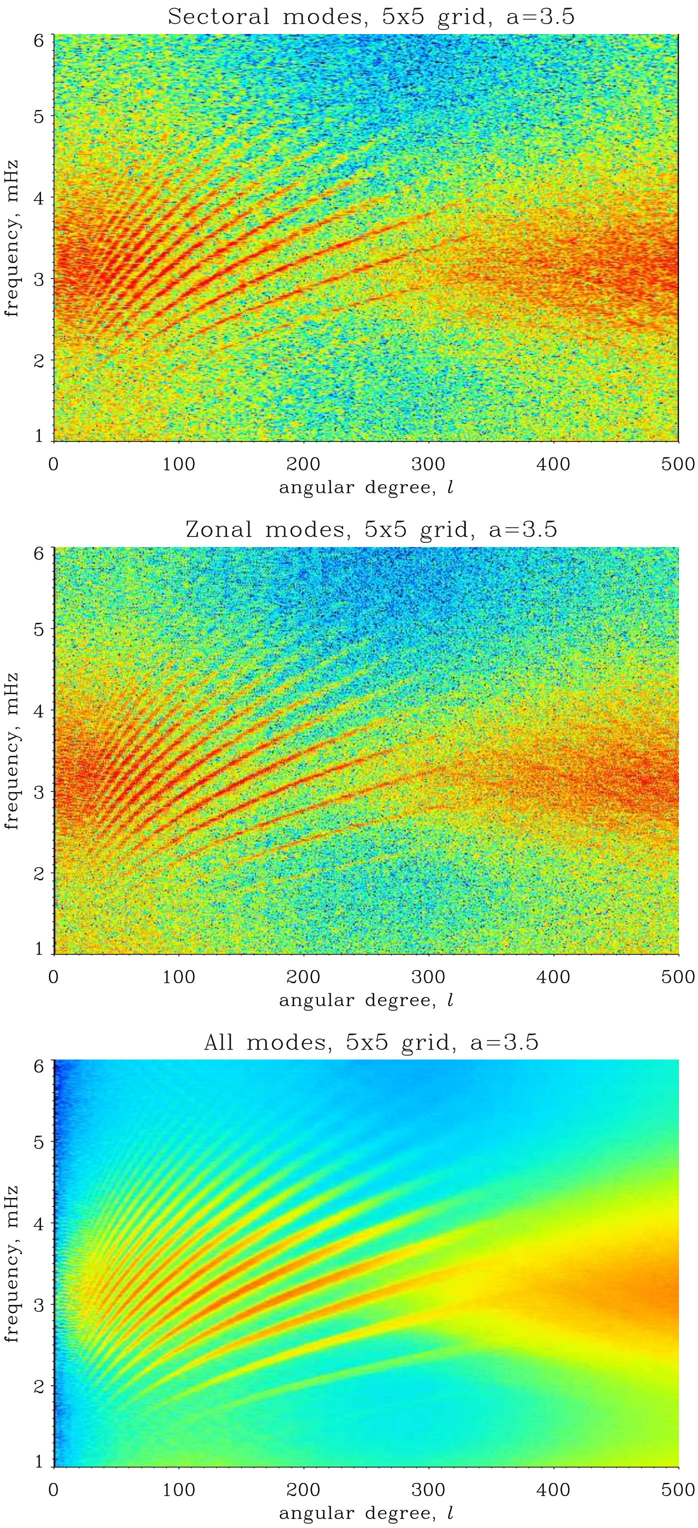

Figure 4: The same as in Fig.3, but for $a = b = 3.5$.

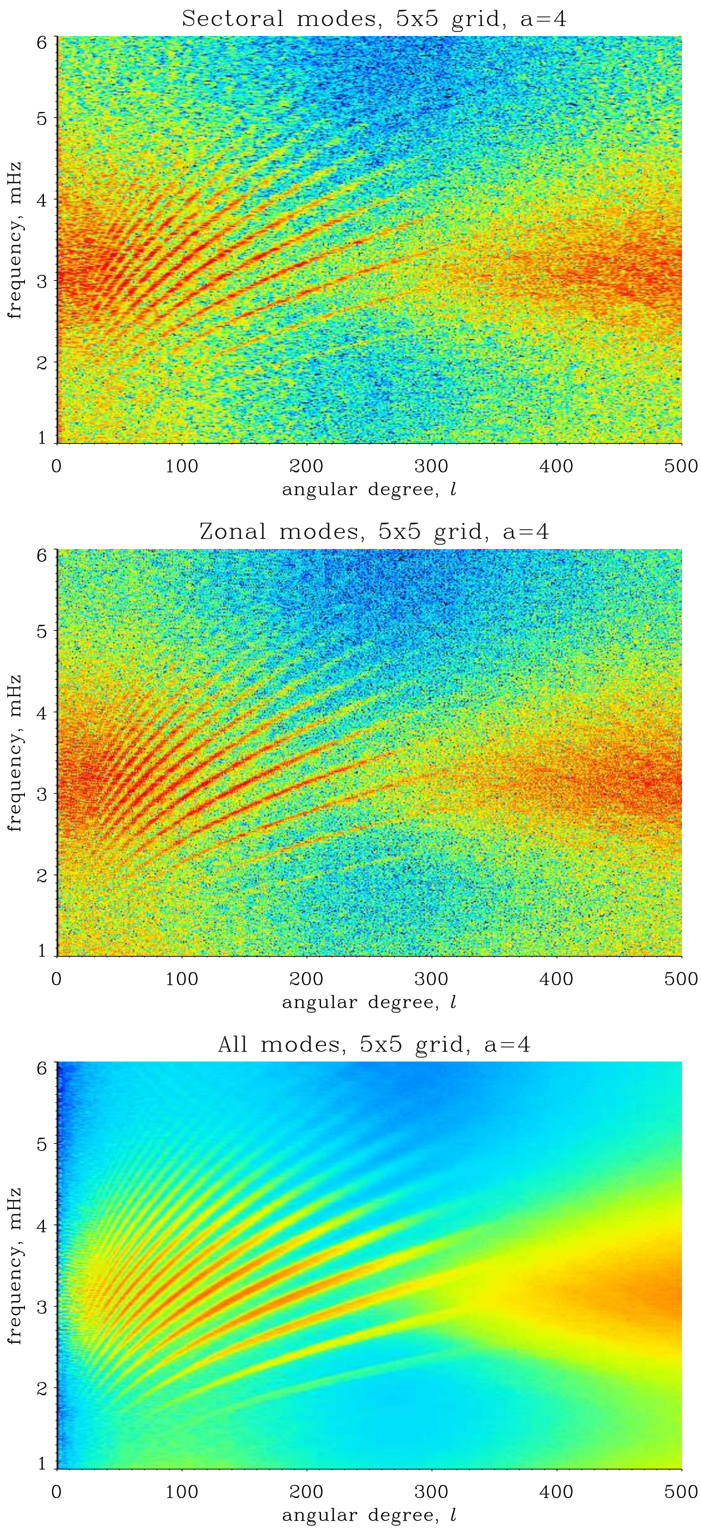

Figure 5: The same as in Fig.3, but for $a = b = 4$.

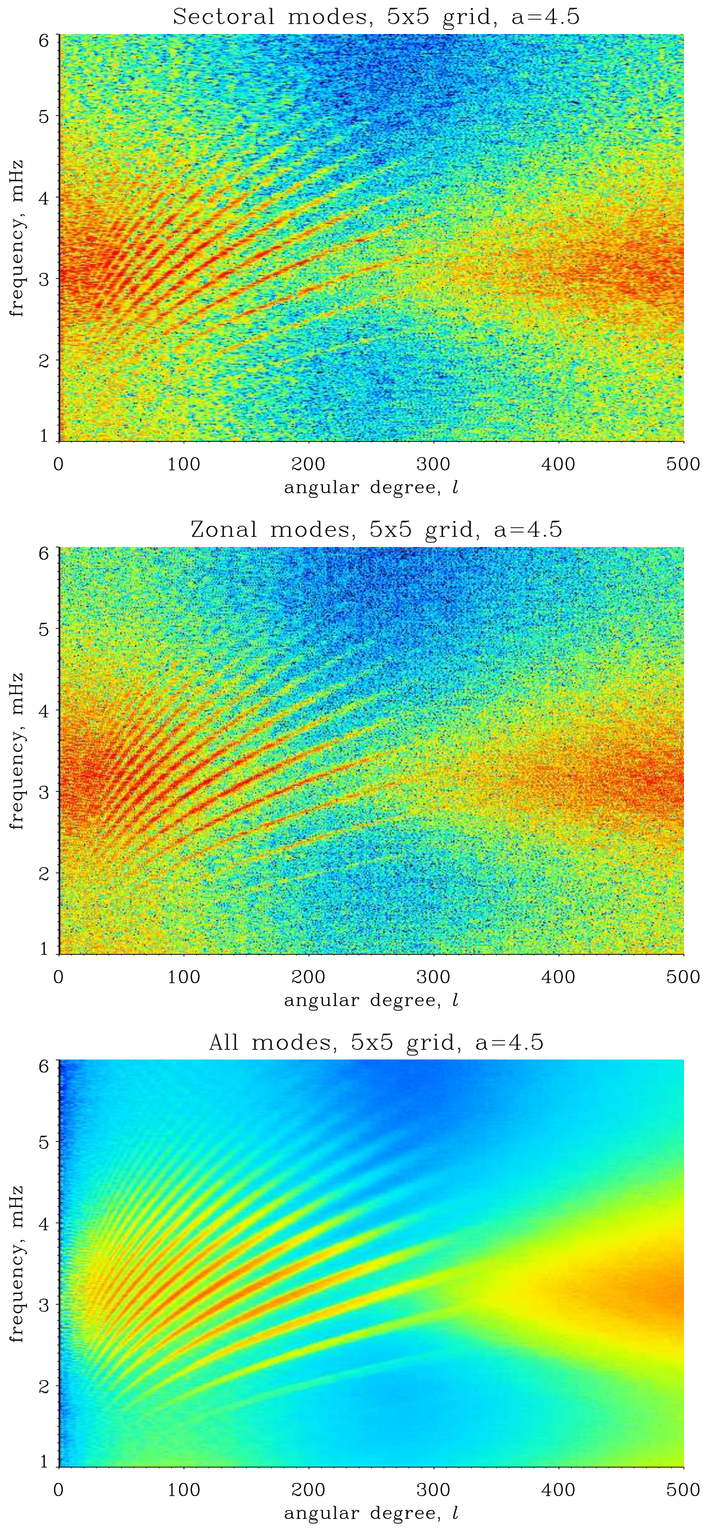

Figure 6: The same as in Fig.3, but for $a = b = 4.5$.

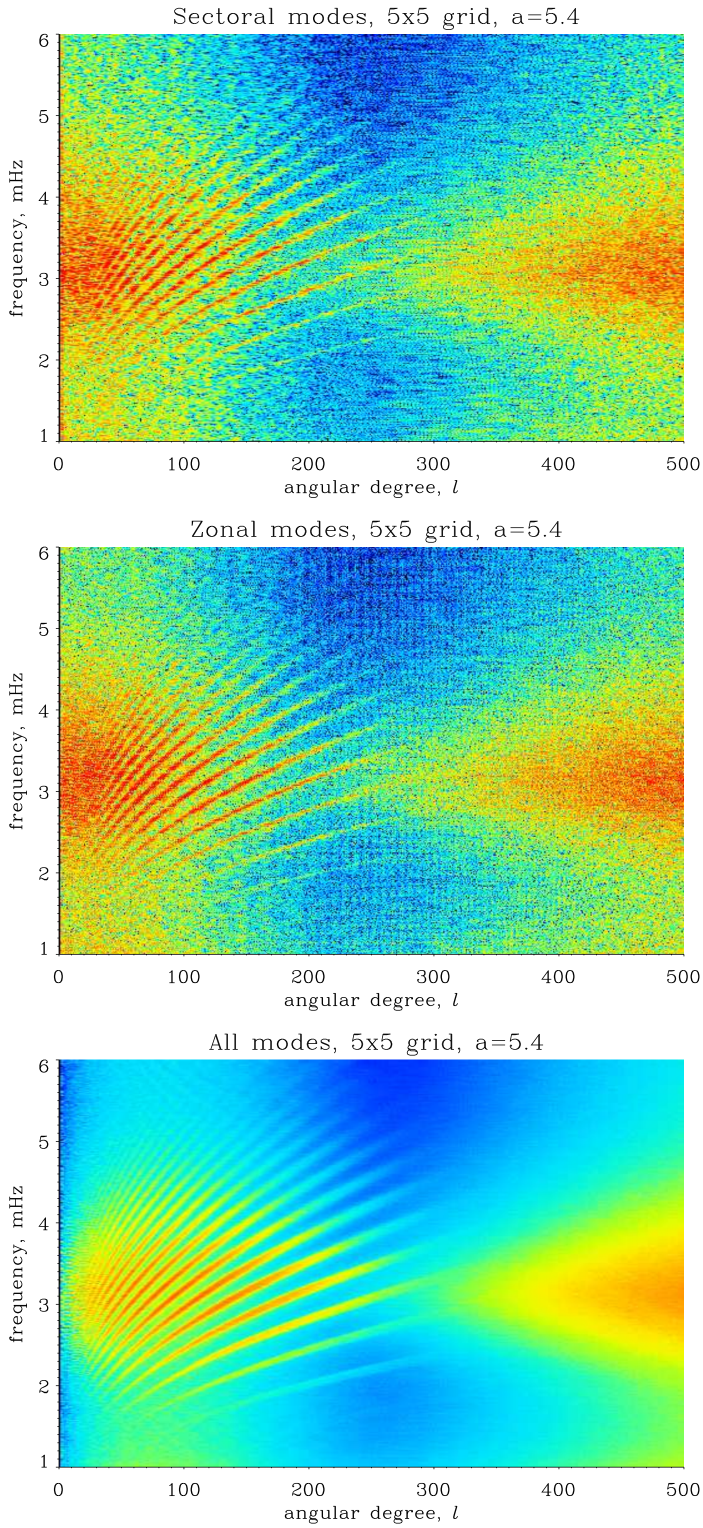

Figure 7: The same as in Fig.3, but for $a = b = 5.4$.

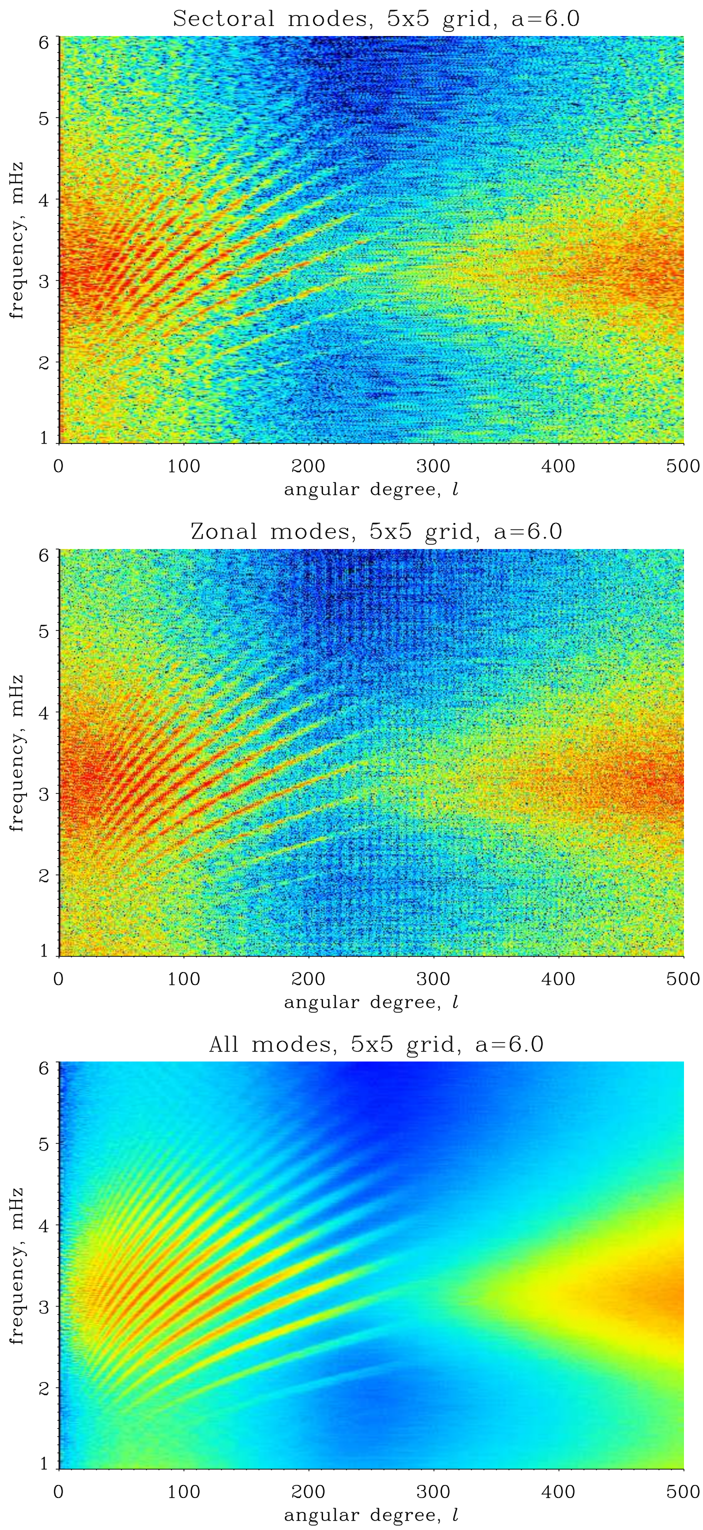

Figure 8: The same as in Fig.3, but for $a = b = 6.0$.